

Examen de Matemáticas 4º de ESO
Diciembre 2010

Resolver las siguientes ecuaciones y sistemas:

Problema 1

$$2\log(x-1) - \log(x+4) = 1$$

Solución:

$$\log\left(\frac{(x-1)^2}{x+4}\right) = \log 10 \implies x^2 - 12x - 39 = 0 \implies$$

$$\begin{cases} x = 14,66 \\ x = -2,66 \text{ No Vale} \end{cases}$$

Problema 2

$$3^{2x-1} - 3^{x+1} - 2 = 0$$

Solución:

$$\frac{(3^x)^2}{3} - 3(3^x) - 2 = 0 \implies \frac{t^2}{3} - 3t - 2 = 0 \implies \begin{cases} t = 9,623 \\ t = -0,623 \end{cases}$$

$$\begin{cases} t = 9,623 = 3^x \implies x = 2,061 \\ t = -0,623 = 3^x \implies \text{No Vale} \end{cases}$$

Problema 3

$$\begin{cases} \log(x^2y) = 6 \\ \log\left(\frac{x}{y^4}\right) = -6 \end{cases}$$

Solución:

$$\begin{cases} \log(x^2y) = 6 \\ \log\left(\frac{x}{y^4}\right) = -6 \end{cases} \implies \begin{cases} 2u + v = 6 \\ u - 4v = -6 \end{cases} \implies$$

$$\begin{cases} u = \log x = 2 \implies x = 100 \\ v = \log y = 2 \implies y = 100 \end{cases}$$

Problema 4

$$\begin{cases} 3^{x+1} - 2 \cdot 5^y = -4 \\ 3^x + 5^{y-1} = 3 \end{cases}$$

Solución:

$$\begin{cases} 3(3^x) - 2(5^y) = -4 \\ 3^x + \frac{5^y}{5} = 3 \end{cases} \implies \begin{cases} 3u - 2v = -4 \\ u + \frac{v}{5} = 3 \end{cases} \implies \begin{cases} u = 2 = 3^x \implies x = 0,631 \\ v = 5 = 5^y \implies y = 1 \end{cases}$$

Problema 5

$$\frac{x+1}{3} - x \leq \frac{x+1}{6} - \frac{x-1}{9}$$

Solución:

$$6x + 6 - 18x \leq 3x + 3 - 2x + 2 \implies x \geq \frac{1}{13} \implies \left[\frac{1}{13}, \infty \right)$$

Problema 6

$$\frac{x^2 + 4x - 5}{x - 2} \geq 0$$

Solución:

$$\frac{x^2 - 8x + 15}{x + 3} = \frac{(x-1)(x+5)}{x-2} \geq 0$$

La solución es: $[-5, 1] \cup (2, \infty)$

Problema 7

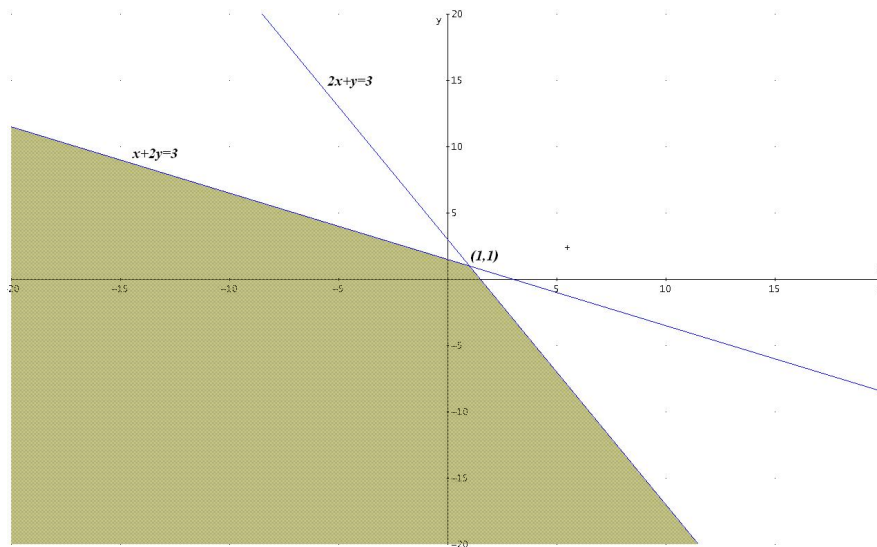
$$\begin{cases} x + 2y \leq 3 \\ 2x + y \leq 3 \end{cases}$$

Solución:

$$x + 2y = 3 \implies \begin{array}{c|c} x & y \\ \hline 0 & 3/2 \\ 3 & 0 \end{array}$$

$$2x + y = 3 \implies \begin{array}{c|c} x & y \\ \hline 0 & 3 \\ 3/2 & 0 \end{array}$$

$$\begin{cases} x + 2y = 3 \\ 2x + y = 3 \end{cases} \implies \begin{cases} x = 1 \\ y = 1 \end{cases} \implies (1, 1)$$



Problema 8

$$\sqrt{2x^2 + 1} = x + 1$$

Solución:

$$\begin{aligned} (\sqrt{2x^2 + 1})^2 &= (x + 1)^2 \implies 2x^2 + 1 = x^2 + 2x + 1 \\ \implies x^2 - 2x &= 0 \implies x = 2, \quad x = 0 \end{aligned}$$

Problema 9

$$\sqrt{x + 3} - \sqrt{x - 2} = 1$$

Solución:

$$\sqrt{x + 3} = 1 + \sqrt{x - 2} \implies x + 3 = 1 + x - 2 + 2\sqrt{x - 2} \implies \sqrt{x - 2} = 2 \implies x = 6$$

Problema 10

$$x^4 - 5x^2 + 4 = 0$$

Solución:

Hacemos $z = x^2 \implies z^2 - 5z + 4 = 0 \implies z = 1$ y $z = 4$.

$$z = 1 = x^2 \implies x = \pm 1$$

$$z = 4 = x^2 \implies x = \pm 2$$