

Examen de Matemáticas 4º de ESO

Diciembre 2010

Resolver las siguientes ecuaciones y sistemas:

Problema 1

$$2 \log x - 1 = \log(x + 3)$$

Solución:

$$\log\left(\frac{x^2}{10}\right) = \log(x + 3) \implies x^2 - 10x - 30 = 0 \implies$$

$$\begin{cases} x = 12,416 \\ x = -2,416 \text{ No Vale} \end{cases}$$

Problema 2

$$2^{2x-1} - 2^{x+1} - 3 = 0$$

Solución:

$$\frac{(2^x)^2}{2} - 2(2^x) - 3 = 0 \implies \frac{t^2}{2} - 2t - 3 = 0 \implies \begin{cases} t = 5,162 \\ t = -1,162 \end{cases}$$

$$\begin{cases} t = 5,162 = 2^x \implies x = 2,368 \\ t = -1,162 = 2^x \implies \text{No Vale} \end{cases}$$

Problema 3

$$\begin{cases} \log(xy^3) = 8 \\ \log\left(\frac{x^2}{y^3}\right) = -2 \end{cases}$$

Solución:

$$\begin{cases} \log(xy^3) = 8 \\ \log\left(\frac{x^2}{y^3}\right) = -2 \end{cases} \implies \begin{cases} u + 3v = 8 \\ 2u - 3v = -2 \end{cases} \implies$$

$$\begin{cases} u = \log x = 2 \implies x = 100 \\ v = \log y = 2 \implies y = 100 \end{cases}$$

Problema 4

$$\begin{cases} 2^{x-1} - 3^{y+1} = -5 \\ 2^{x+2} - 3^y = 6 \end{cases}$$

Solución:

$$\begin{cases} \frac{2^x}{2} - 3(3^y) = -5 \\ 4(2^x) - 3^y = 6 \end{cases} \implies \begin{cases} \frac{u}{2} - 3v = -5 \\ 4u - v = 6 \end{cases} \implies \begin{cases} u = 2 = 3^x \implies x = 1 \\ v = 2 = 5^y \implies y = 0,631 \end{cases}$$

Problema 5

$$\frac{x-3}{4} - x \geq 1 - \frac{x-1}{5}$$

Solución:

$$5x - 15 - 20x \geq 20 - 4x + 4 \implies x \leq -\frac{39}{11} \implies \left(-\infty, -\frac{39}{11}\right]$$

Problema 6

$$\frac{x^2 - 8x + 15}{x + 3} \geq 0$$

Solución:

$$\frac{x^2 - 8x + 15}{x + 3} = \frac{(x-3)(x-5)}{x+3} \geq 0$$

La solución es: $(-3, 3] \cup [5, \infty)$

Problema 7

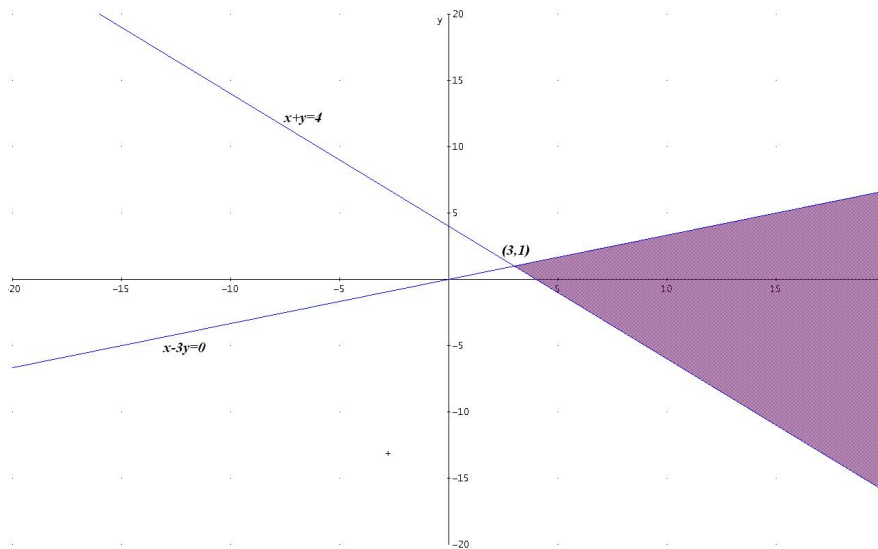
$$\begin{cases} x + y \geq 4 \\ x - 3y \geq 0 \end{cases}$$

Solución:

$$x + y = 4 \implies \begin{array}{c|c} x & y \\ \hline 0 & 4 \\ 4 & 0 \end{array}$$

$$x - 3y = 0 \implies \begin{array}{c|c} x & y \\ \hline 0 & 0 \\ 3 & 1 \end{array}$$

$$\begin{cases} x + y = 4 \\ x - 3y = 0 \end{cases} \implies \begin{cases} x = 3 \\ y = 1 \end{cases} \implies (3, 1)$$



Problema 8

$$\sqrt{5x^2 - 1} = 3x - 1$$

Solución:

$$\begin{aligned} (\sqrt{5x^2 - 1})^2 &= (3x - 1)^2 \implies 5x^2 - 1 = 9x^2 - 6x + 1 \\ \implies 4x^2 - 6x + 2 &= 0 \implies x = 1, \quad x = \frac{1}{2} \end{aligned}$$

Problema 9

$$\sqrt{x+2} - \sqrt{x-3} = 1$$

Solución:

$$\sqrt{x+2} = 1 + \sqrt{x-3} \implies x+2 = 1+x-3+2\sqrt{x-3} \implies \sqrt{x-3} = 2 \implies x = 7$$

Problema 10

$$x^4 - 7x^2 - 18 = 0$$

Solución:

Hacemos $z = x^2 \implies z^2 + 7z - 18 = 0 \implies z = 9$ y $z = -2$.

$$z = 9 = x^2 \implies x = \pm 3$$

$$z = -2 = x^2 \text{ No Vale}$$