Examen de Matemáticas 4º de ESO

Noviembre 2009

Resolver las siguientes ecuaciones y sistemas:

Problema 1

$$\log(3x+1) - 1 = 2\log x$$

Solución:

$$\log\left(\frac{3x+1}{10}\right) = \log x^2 \Longrightarrow 10x^2 - 3x - 1 = 0 \Longrightarrow$$

$$\begin{cases} x = 1/2 \\ x = -1/5 \text{ no vale} \end{cases}$$

Problema 2

$$2 \cdot 3^{2x} + 3^{x-1} - 1 = 0$$

Solución:

$$2(3^x)^2 + \frac{3^x}{3} - 1 = 0 \Longrightarrow 6t^2 + t - 3 = 0 \Longrightarrow \begin{cases} t = 0,6286669787 \\ t = -0,7953336454 \end{cases}$$

$$\begin{cases} t = 0,6286669787 = 3^x \Longrightarrow x = -0,4224908211 \\ t = -0,7953336454 = 3^x \Longrightarrow \text{ No Vale} \end{cases}$$

Problema 3

$$\begin{cases} \log(x^2 y) & = 13 \\ \log\left(\dfrac{x^2}{y}\right) & = 7 \end{cases}$$

Solución:

$$\begin{cases} 2\log x + & \log y = & 13 \\ 2\log x - & \log y = & 7 \end{cases} \Longrightarrow \begin{cases} 2u + & v = & 13 \\ 2u - & v = & 7 \end{cases} \Longrightarrow$$

$$\begin{cases} u = \log x = 5 \Longrightarrow x = 10^5 \\ v = \log y = 3 \Longrightarrow y = 10^3 \end{cases}$$

Problema 4

$$\begin{cases} 2^{x+1} - & 3^{y-1} & = 1 \\ 2^x + & 3^y & = 4 \end{cases}$$

Solución:

$$\left\{\begin{array}{rrl} 2\cdot 2^x- & \dfrac{3^y}{3} & =1 \\ & & \\ 2^x+ & 3^y & =4 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrl} 2u- & \dfrac{v}{3} & =1 \\ & & \\ u+ & v & =4 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u=1=3^x \Longrightarrow x=0 \\ \\ v=3=3^y \Longrightarrow y=1 \end{array}\right.$$

Problema 5

$$\frac{x-2}{5}+1 \geq \frac{x-1}{2}-\frac{1-x}{10}$$

Solución:

$$2x+6 \geq 6x-6 \Longrightarrow -4x \geq -12 \Longrightarrow x \leq 3 \Longrightarrow (-\infty, 3]$$

Problema 6

$$\frac{x^2-2x-15}{x+1} \leq 0$$

Solución:

$$\frac{x^2-2x-15}{x+1}=\frac{(x-5)(x+3)}{x+1} \leq 0$$

	$(-\infty,-3)$	$(-3,-1)$	$(-1,5)$	$(5,\infty)$
$f(x)$	$-$	$+$	$-$	$+$

La solución es: $(-\infty,-3] \cup (-1,5]$

Problema 7

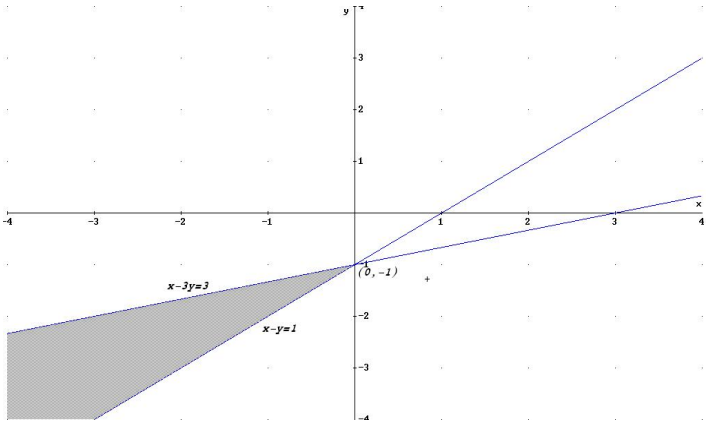
$$\left\{\begin{array}{rrl} x- & 3y & \geq 3 \\ x- & y & <1 \end{array}\right.$$

Solución:

$$x-3y=3 \Longrightarrow \begin{array}{c|c} x & y \\ \hline 0 & -1 \\ 3 & 0 \end{array}$$

$$x-y=1 \Longrightarrow \begin{array}{c|c} x & y \\ \hline 0 & -1 \\ 1 & 0 \end{array}$$

$$\left\{\begin{array}{rrl} x- & 3y & =3 \\ x- & y & =1 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x=0 \\ y=-1 \end{array}\right. \Longrightarrow (0,-1)$$



Problema 8

$$\sqrt{5x+1}-1=x$$

Solución:

$$5x+1=x^2+2x+1 \Longrightarrow x^2-3x=0 \Longrightarrow x=0,\ \ x=3$$

Problema 9

$$\sqrt{x+3}-\sqrt{x-2}=1$$

Solución:

$$\sqrt{x+3}=1+\sqrt{x-2} \Longrightarrow x+3=1+x-2-2\sqrt{x-2} \Longrightarrow 2=-\sqrt{x-2} \Longrightarrow x=6$$

Problema 10

$$x^4-15x^2-16=0$$

Solución:

Hacemos $z=x^2 \Longrightarrow z^2-15z-16=0 \Longrightarrow z=16$ y $z=-1$.

$$z=16=x^2 \Longrightarrow x=\pm 4$$

$$z=-1=x^2 \text{ No Vale}$$