

Examen de Matemáticas 4º de ESO

Noviembre 2007

Resolver las siguientes ecuaciones y sistemas:

Problema 1 Resolver:

1. $\log(x - 1) + \log x = 1$
2. $\log(1 - x) - 2 = 2 \log x$

Solución:

1. $\log(x - 1) + \log x = 1 \implies x^2 - x - 10 = 0 \implies x = 3,7 \text{ y } x = -2,7$
No Vale
2. $\log(1 - x) - 2 = 2 \log x \implies 100x^2 + x - 1 = 0 \implies x = 0,095 \text{ y } x = -0,105$ No vale

Problema 2

$$3^{2x-1} - 3^{x+1} - 2 = 0$$

Solución:

$$3^{2x-1} - 3^{x+1} - 2 = 0 \implies \frac{t^2}{3} - 3t - 2 = 0 \implies t^2 - 9t - 6 = 0$$

$$\begin{cases} t = 3^x = -0,6234753829 \implies \text{No Vale} \\ t = 3^x = 9,623475382 \implies x = 2,060968632 \end{cases}$$

Problema 3

$$\begin{cases} \log(x^2y^2) = 8 \\ \log\left(\frac{x^2}{y}\right) = 5 \end{cases}$$

Solución:

$$\begin{cases} 2\log x + 2\log y = 8 \\ 2\log x - \log y = 5 \end{cases} \implies \begin{cases} 2u + 2v = 8 \\ 2u - v = 5 \end{cases} \implies$$
$$\begin{cases} u = \log x = 3 \implies x = 1000 \\ v = \log y = 1 \implies y = 10 \end{cases}$$

Problema 4

$$\begin{cases} 5^x - 2 \cdot 3^y = 2 \\ 2 \cdot 5^x + 3^y = 10 \end{cases}$$

Solución:

$$\begin{cases} 5^x - 2 \cdot 3^y = 2 \\ 2 \cdot 5^x + 3^y = 10 \end{cases} \implies \begin{cases} u - 2v = 2 \\ 2u + v = 10 \end{cases} \implies$$

$$\begin{cases} u = 4, 4 = 5^x \implies x = 0, 92 \\ v = 1, 2 = 3^y \implies y = 1, 349 \end{cases}$$

Problema 5

$$\frac{x}{3} - \frac{x+1}{6} \geq 1 - \frac{x}{2}$$

Solución:

$$\frac{x}{3} - \frac{x+1}{6} \geq 1 - \frac{x}{2} \implies x \geq \frac{7}{4} \implies \left[\frac{7}{4}, +\infty \right)$$

Problema 6

$$\frac{x^2 - 5x - 14}{x - 1} \geq 0$$

Solución:

$$\frac{x^2 - 5x - 14}{x - 1} = \frac{(x - 7)(x + 2)}{x - 1} \geq 0$$

La solución es: $[-2, 1) \cup [7, \infty)$

Problema 7

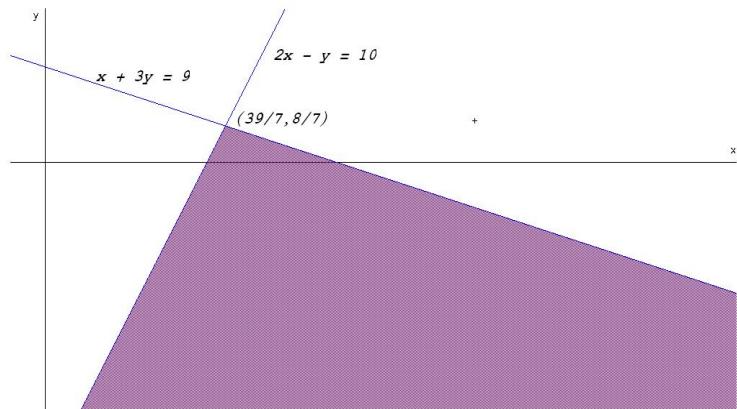
$$\begin{cases} 2x - y \geq 10 \\ x + 3y \leq 9 \end{cases}$$

Solución:

$$2x - y = 10 \implies \begin{array}{c|c} x & y \\ \hline 0 & -10 \\ 5 & 0 \end{array}$$

$$x + 3y = 9 \implies \begin{array}{c|c} x & y \\ \hline 0 & 3 \\ 9 & 0 \end{array}$$

$$\begin{cases} 2x - y = 10 \\ x + 3y = 9 \end{cases} \quad \begin{cases} x = 39/7 \\ y = 8/7 \end{cases} \implies \left(\frac{39}{7}, \frac{8}{7} \right)$$



Problema 8

$$\sqrt{2x+1} - \sqrt{x-4} = 3$$

Solución:

$$\sqrt{2x+1} - \sqrt{x-4} = 3 \implies 2x+1 = 9 + (x-4) + 6\sqrt{x-4} \implies x-4 = 6\sqrt{x-2}$$

$$x^2 - 8x + 16 = 36x - 144 \implies x^2 - 44x + 160 = 0$$

$$\begin{cases} x = 4 \\ x = 40 \end{cases}$$

Problema 9

$$\sqrt{5x-1} - x = 1$$

Solución:

$$5x - 1 = 1 + x^2 + 2x \implies x^2 - 3x + 2 = 0$$

$$\begin{cases} x = 1 \\ x = 2 \end{cases}$$

Problema 10

$$x^4 - 26x^2 + 25 = 0$$

Solución:

$$\text{Hacemos } z = x^2 \implies z^2 - 26z + 25 = 0 \implies z = 25 \text{ y } z = 1.$$

$$z = 25 = x^2 \implies x = \pm 5$$

$$z = 1 = x^2 \implies x = \pm 1$$