

Examen de Matemáticas 4º de ESO
Noviembre 2006

Resolver las siguientes ecuaciones y sistemas:

Problema 1

$$\log(x+1) - \log(x^2-1) = 1$$

Solución:

$$\log\left(\frac{x+1}{x^2-1}\right) = \log 10 \implies 10x^2 - x - 11 = 0 \implies \begin{cases} x = 1, 1 \\ x = -1 \text{ No Vale} \end{cases}$$

Problema 2

$$5^{2x-1} - 5^x + 1 = 0$$

Solución:

$$\frac{(5^x)^2}{5} - 5^x + 1 = 0 \implies \frac{t^2}{5} - t + 1 = 0 \implies t^2 - 5t + 5 = 0$$
$$\begin{cases} t = 5^x = 3,618 \implies x = 0,714 \\ t = 5^x = 1,381 \implies x = 0,296 \end{cases}$$

Problema 3

$$\begin{cases} \log(x^3y^2) = 8 \\ \log\left(\frac{x}{y}\right) = 1 \end{cases}$$

Solución:

$$\begin{cases} 3\log x + 2\log y = 8 \\ \log x - \log y = 1 \end{cases} \implies \begin{cases} 3u + 2v = 8 \\ u - v = 1 \end{cases} \implies$$
$$\begin{cases} u = \log x = 2 \implies x = 100 \\ v = \log y = 1 \implies y = 10 \end{cases}$$

Problema 4

$$\begin{cases} 2^{x+1} - 3^y = 1 \\ 2^x + 2 \cdot 3^y = 3 \end{cases}$$

Solución:

$$\begin{cases} 2 \cdot 2^x - 3^y = 1 \\ 2^x + 2 \cdot 3^y = 3 \end{cases} \implies \begin{cases} 2u - v = 1 \\ u + 2v = 3 \end{cases} \implies$$
$$\begin{cases} u = 1 = 2^x \implies x = 0 \\ v = 1 = 3^y \implies y = 0 \end{cases}$$

Problema 5

$$\frac{x+2}{12} - \frac{x+1}{4} \leq 1 + \frac{x}{3}$$

Solución:

$$x+2-3x \leq 12+4x \implies -6x \leq 13 \implies x \geq -\frac{13}{6} \implies \left[-\frac{13}{6}, +\infty\right)$$

Problema 6

$$\frac{x^2 - 2x - 3}{x - 1} \geq 0$$

Solución:

$$\frac{x^2 - 2x - 3}{x - 1} = \frac{(x+1)(x-3)}{x-1} \geq 0$$

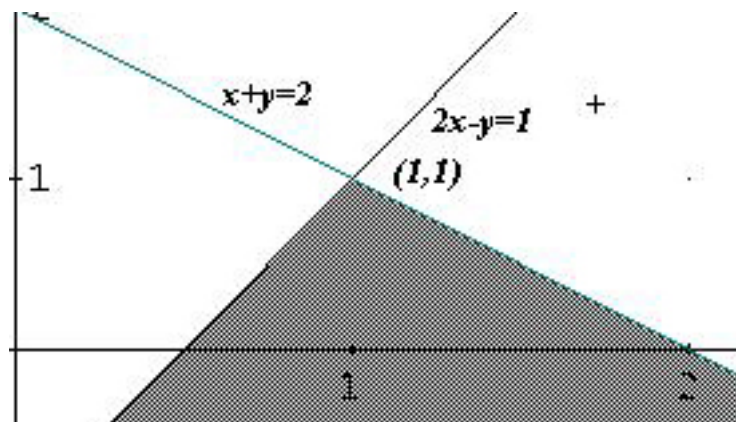
	$(-\infty, -1)$	$(-1, 1)$	$(1, 3)$	$(3, \infty)$
$x+1$	-	+	+	+
$x-1$	-	-	+	+
$x-3$	-	-	-	+
$\frac{x^2-2x-3}{x-1}$	-	+	-	+

La solución es: $[-1, 1) \cup [3, \infty)$

Problema 7

$$\begin{cases} 2x - y > 1 \\ x + y < 2 \end{cases}$$

Solución:



$$2x - y = 1 \implies \begin{array}{c|c} x & y \\ \hline 0 & -1 \\ 1/2 & 0 \end{array}$$

$$x + y = 2 \implies \begin{array}{c|c} x & y \\ \hline 0 & 2 \\ 2 & 0 \end{array}$$

$$\begin{cases} 2x - y = 1 \\ x + y = 2 \end{cases} \implies \begin{cases} x = 1 \\ y = 1 \end{cases} \implies (1, 1)$$

Problema 8

$$\sqrt{2x+3} - \sqrt{x-2} = 2$$

Solución:

$$\sqrt{2x+3} = 2 + \sqrt{x-2} \implies 2x+3 = 4 + x - 2 + 4\sqrt{x-2} \implies x+1 = 4\sqrt{x-2}$$

$$x^2 + 2x + 1 = 16x + 32 \implies x^2 - 14x + 33 = 0$$

$$\begin{cases} x = 3 \\ x = 11 \end{cases}$$

Problema 9

$$\sqrt{3x-5} + x = 1$$

Solución:

$$3x - 5 = 1 + x^2 - 2x \implies x^2 - 5x + 6 = 0$$

$$\begin{cases} x = 3 \text{ No Vale} \\ x = 2 \text{ No Vale} \end{cases}$$

Problema 10

$$x^4 - 24x^2 - 25 = 0$$

Solución:

$$\text{Hacemos } z = x^2 \implies z^2 - 24z - 25 = 0 \implies z = 25 \text{ y } z = -1.$$

$$z = 25 = x^2 \implies x = \pm 5$$

$$z = -1 = x^2 \text{ No Vale}$$