Examen de Matemáticas 4º de ESO

Noviembre 2005

Resolver las siguientes ecuaciones y sistemas:

Problema 1

$$2\log x - \log(x-1) = 1$$

Solución:

$$\log\left(\frac{x^2}{x-1}\right) = \log 10 \Longrightarrow x^2 - 10x + 10 = 0 \Longrightarrow$$

$$\begin{cases} x = 1,127016653 \\ x = 8,872983346 \end{cases}$$

Problema 2

$$2^{2x-2} - 2^{x-1} - 1 = 0$$

Solución:

$$\frac{(2^x)^2}{2^2} - \frac{2^x}{2} - 1 = 0 \Longrightarrow t^2 - 2t - 4 = 0 \Longrightarrow \begin{cases} t = 3,236067977 \\ t = -1,236067977 \end{cases}$$

$$\begin{cases} t = 3,236067977 = 5^x \Longrightarrow x = 1,694241913 \\ t = -1,236067977 = 5^x \Longrightarrow \text{ No Vale} \end{cases}$$

Problema 3

$$\begin{cases} \log(x^2y^3) & = 6 \\ \log\left(\dfrac{x^2}{y^2}\right) & = 1 \end{cases}$$

Solución:

$$\begin{cases} 2\log x + & 3\log y = & 6 \\ 2\log x - & 2\log y = & 1 \end{cases} \Longrightarrow \begin{cases} 2u + & 3v = & 6 \\ 2u - & 2v = & 1 \end{cases} \Longrightarrow$$

$$\begin{cases} u = \log x = 3/2 \Longrightarrow x = 10^{3/2} \\ v = \log y = 1 \Longrightarrow y = 10 \end{cases}$$

Problema 4

$$\begin{cases} 3^{x-2} - & 5^y & = -2 \\ 3^x + & 5^y & = 12 \end{cases}$$

Solución:

$$\left\{\begin{array}{rcl} \frac{3^x}{9} - & 5^y & = -2 \\ 3^x + & 5^y & = 12 \end{array}\right. \Longrightarrow \left\{\begin{array}{rcl} \frac{u}{9} - & v & = -2 \\ u + & v & = 12 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u = 9 = 3^x \Longrightarrow x = 2 \\ v = 3 = 5^y \Longrightarrow y = 0,6826061944 \end{array}\right.$$

Problema 5

$$\frac{5-3x}{8} - \frac{x}{12} \leq 1 - \frac{x+2}{6}$$

Solución:

$$15 - 9x - 2x \leq 24 - 4x - 8 \Longrightarrow -7x \leq 1 \Longrightarrow x \geq -\frac{1}{7} \Longrightarrow \left[-\frac{1}{7}, +\infty\right)$$

Problema 6

$$\frac{x^2 - 6x - 16}{x - 1} \geq 0$$

Solución:

$$\frac{x^2 - 6x - 16}{x - 1} = \frac{(x-8)(x+2)}{x-1} \leq 0$$

	$(-\infty, -2)$	$(-2, 1)$	$(1, 8)$	$(8, \infty)$
$x + 2$	$-$	$+$	$+$	$+$
$x - 1$	$-$	$-$	$+$	$+$
$x - 8$	$-$	$-$	$-$	$+$
$\frac{x^2-6x-16}{x-1}$	$-$	$+$	$-$	$+$

La solución es: $[-2, 1) \cup [8, +\infty)$

Problema 7

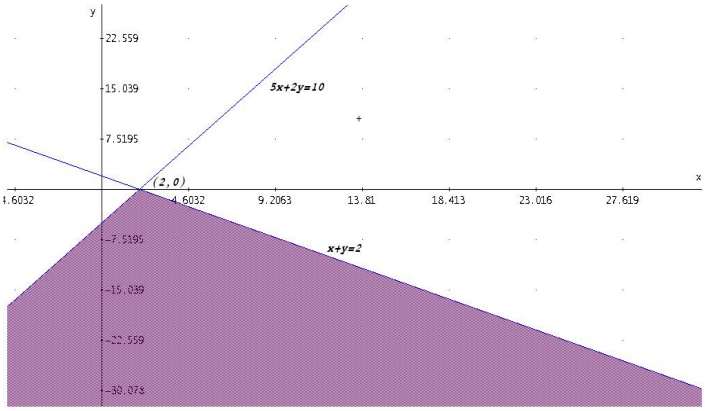
$$\left\{\begin{array}{rcl} 5x - & 2y & \geq 10 \\ x + & y & < 2 \end{array}\right.$$

Solución:

$$5x - 2y = 10 \Longrightarrow \begin{array}{c|c} x & y \\ \hline 0 & -5 \\ 2 & 0 \end{array}$$

$$x+y=2 \Longrightarrow \begin{array}{c|c} x & y \\ \hline 0 & 2 \\ 2 & 0 \end{array}$$

$$\left\{\begin{array}{rrr} 5x- & 2y & =10 \\ x+ & y & =2 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x=2 \\ y=0 \end{array}\right. \Longrightarrow (2,0)$$



Problema 8

$$\sqrt{9-2x}=x-3$$

Solución:

$$9-2x=x^2-6x+9 \Longrightarrow x^2-4x=0 \Longrightarrow x=4,\ x=0\,(\text{No vale})$$

Problema 9

$$\sqrt{x+5}-\sqrt{x}=2$$

Solución:

$$\sqrt{x+5}=2+\sqrt{x} \Longrightarrow x+5=4+x+4\sqrt{x} \Longrightarrow 1=4\sqrt{x} \Longrightarrow x=\frac{1}{16}$$

Problema 10

$$x^4-24x^2-25=0$$

Solución:

Hacemos $z=x^2 \Longrightarrow z^2-24z-25=0 \Longrightarrow z=25\ y\ z=-1.$

$$z=25=x^2 \Longrightarrow x=\pm 5$$

$$z=-1=x^2 \text{ No Vale}$$