Examen de Matemáticas 4º de ESO

Noviembre 2005

Resolver las siguientes ecuaciones y sistemas:

Problema 1

$$\log(2x+2) - 1 = 2\log x$$

Solución:

$$\log\left(\frac{2x+2}{10}\right) = \log x^2 \Longrightarrow 5x^2 - x - 1 = 0 \Longrightarrow$$

$$\begin{cases} x = 0.5582575694 \\ x = -0.3582575694 \text{ no vale} \end{cases}$$

Problema 2

$$3 \cdot 5^{2x} + 5^{x-1} - 1 = 0$$

Solución:

$$3(5^x)^2 + \frac{5^x}{5} - 1 = 0 \Longrightarrow 15t^2 + t - 5 = 0 \Longrightarrow \begin{cases} t = 0,5449783857 \\ t = -0,6116450524 \end{cases}$$

$$\begin{cases} t = 0,5449783857 = 5^x \Longrightarrow x = x = -0,3771559869 \\ t = -0,6116450524 = 5^x \Longrightarrow \text{ No Vale} \end{cases}$$

Problema 3

$$\begin{cases} \log(x^2 y) & = 5 \\ \log\left(\dfrac{x^2}{y}\right) & = 3 \end{cases}$$

Solución:

$$\begin{cases} 2\log x + & \log y = & 5 \\ 2\log x - & \log y = & 3 \end{cases} \Longrightarrow \begin{cases} 2u + & v = & 5 \\ 2u - & v = & 3 \end{cases} \Longrightarrow$$

$$\begin{cases} u = \log x = 2 \Longrightarrow x = 100 \\ v = \log y = 1 \Longrightarrow y = 10 \end{cases}$$

Problema 4

$$\begin{cases} 3^{x+1} - & 2^{y-1} & = 3 \\ 3^x + & 2^y & = 15 \end{cases}$$

Solución:

$$\left\{\begin{array}{rrrl} 3\cdot 3^x- & \dfrac{2^y}{2} & = 3 \\ \\ 3^x+ & 2^y & = 15 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrrl} 3u- & \dfrac{v}{2} & = 3 \\ \\ u+ & v & = 15 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u = 3 = 3^x \Longrightarrow x = 1 \\ \\ v = 12 = 3^y \Longrightarrow y = 3,584962500 \end{array}\right.$$

Problema 5

$$\frac{x-3}{5} + 2 \geq \frac{x}{2} - \frac{1-x}{10}$$

Solución:

$$2x + 14 \geq 6x - 1 \Longrightarrow -4x \geq -15 \Longrightarrow x \leq \frac{15}{4} \Longrightarrow \left(-\infty, \frac{15}{4}\right]$$

Problema 6

$$\frac{x^2+3x-18}{x-1} \leq 0$$

Solución:

$$\frac{x^2+3x-18}{x-1} = \frac{(x+6)(x-3)}{x-1} \leq 0$$

	$(-\infty,-6)$	$(-6,1)$	$(1,3)$	$(3,\infty)$
$x+6$	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$+$	$+$
$x-3$	$-$	$-$	$-$	$+$
$\frac{x^2+3x-18}{x-1}$	$-$	$+$	$-$	$+$

La solución es: $(-\infty,-6] \cup (1,3]$

Problema 7

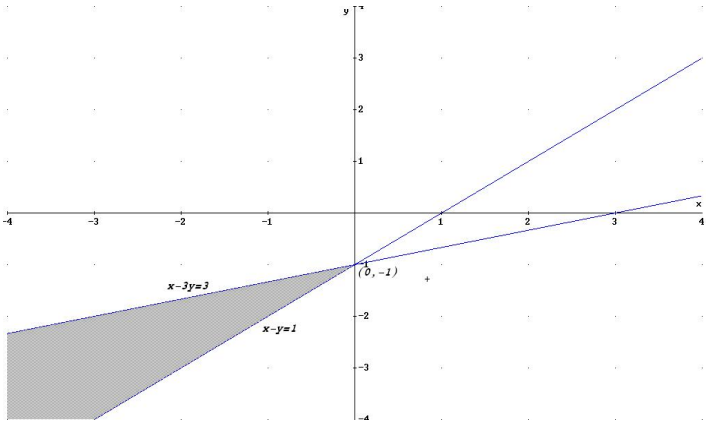
$$\left\{\begin{array}{rrl} x- & 3y & \geq 3 \\ x- & y & < 1 \end{array}\right.$$

Solución:

$$x - 3y = 3 \Longrightarrow \begin{array}{c|c} x & y \\ \hline 0 & -1 \\ 3 & 0 \end{array}$$

$$x - y = 1 \Longrightarrow \begin{array}{c|c} x & y \\ \hline 0 & -1 \\ 1 & 0 \end{array}$$

$$\left\{ \begin{array}{rrl} x- & 3y & = 3 \\ x- & y & = 1 \end{array} \right. \Longrightarrow \left\{ \begin{array}{l} x = 0 \\ y = -1 \end{array} \right. \Longrightarrow (0, -1)$$



Problema 8

$$\sqrt{5x + 4} = x + 2$$

Solución:

$$5x + 4 = x^2 + 4x + 4 \Longrightarrow x^2 - x = 0 \Longrightarrow x = 0, \;\; x = 1$$

Problema 9

$$\sqrt{x + 2} - \sqrt{x} = 1$$

Solución:

$$\sqrt{x + 2} = 1 + \sqrt{x} \Longrightarrow x + 2 = 1 + x + 2\sqrt{x} \Longrightarrow 1 = 2\sqrt{x} \Longrightarrow x = \frac{1}{4}$$

Problema 10

$$x^4 - 8x^2 - 9 = 0$$

Solución:

Hacemos $z = x^2 \Longrightarrow z^2 - 8z - 9 = 0 \Longrightarrow z = 9 \; y \; z = -1.$

$$z = 9 = x^2 \Longrightarrow x = \pm 3$$

$$z = -1 = x^2 \text{ No Vale}$$