Examen de Matemáticas 4º de ESO

Enero 2005

Resolver las siguientes ecuaciones y sistemas:

Problema 1

$$\log x - \log(1-x) = 2$$

Solución:

$$\log\left(\frac{x}{1-x}\right) = \log 100 \Longrightarrow 101x = 100 \Longrightarrow x = \frac{100}{101}$$

Problema 2

$$2^x - 2^{x-1} - 1 = 0$$

Solución:

$$2^x - \frac{2^x}{2} - 1 = 0 \Longrightarrow t - \frac{t}{2} - 1 = 0 \Longrightarrow\ t = 2 \Longrightarrow 2^x = 2 \Longrightarrow x = 1$$

Problema 3

$$\left\{\begin{array}{rr} \log x + & 2\log y = 3 \\ -\log x + & \log y = 0 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rr} \log x + & 2\log y = 3 \\ -\log x + & \log y = 0 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrr} u + & 2v = & 3 \\ -u + & v = & 0 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u = \log x = 1 \Longrightarrow x = 10 \\ v = \log y = 1 \Longrightarrow y = 10 \end{array}\right.$$

Problema 4

$$\left\{\begin{array}{rrr} 2^x + & 3^y & = 2 \\ 2^{x+1} - & 3^y & = 1 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrr} 2^x + & 3^y & = 2 \\ 2\cdot 2^x - & 3^y & = 1 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrr} u + & v & = 2 \\ & & \\ 2u - & v & = 1 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u = 1 = 2^x \Longrightarrow x = 0 \\ \\ v = 1 = 3^y \Longrightarrow y = 0 \end{array}\right.$$

Problema 5

$$\frac{x-1}{8} - \frac{x}{2} < \frac{x+1}{4}$$

Solución:

$$x - 1 - 4x < 2x + 2 \Longrightarrow -5x < 3 \Longrightarrow x > -\frac{3}{5} \Longrightarrow \left(-\frac{3}{5}, +\infty\right)$$

Problema 6

$$\frac{x^2 - x - 2}{x + 3} \geq 0$$

Solución:

$$\frac{x^2 - x - 2}{x + 3} = \frac{(x+1)(x-2)}{x+3} \geq 0$$

	$(-\infty, -3)$	$(-3, -1)$	$(-1, 2)$	$(2, \infty)$
$x + 3$	$-$	$+$	$+$	$+$
$x + 1$	$-$	$-$	$+$	$+$
$x - 2$	$-$	$-$	$-$	$+$
$\frac{x^2-x-2}{x+3}$	$-$	$+$	$-$	$+$

La solución es: $(-3, -1] \cup [2, \infty)$

Problema 7

$$\sqrt{x^2 - 8} = x + 2$$

Solución:

$$x^2 - 8 = x^2 + 4x + 4 \Longrightarrow x = -3$$

Problema 8

$$x^4 + x^2 - 20 = 0$$

Solución:

Hacemos $z = x^2 \Longrightarrow z^2 + z - 20 = 0 \Longrightarrow z = 4$ *y* $z = -5$.

$$z = 4 = x^2 \Longrightarrow x = \pm 2$$

$$z = -5 = x^2 \text{ No Vale}$$