Examen de Matemáticas 2º de Bachillerato CN
Enero 2024

Problema 1 (2,5 puntos) Se considera la función $f(x) = xe^{2x^2}$. Se pide:

a) (1,5 puntos) Calcula una primitiva de $f(x)$, que pase por el punto $(0, -1)$. (Sugerencia: Puedes utilizar el cambio de variable $t = 2x^2$)

b) (1 punto) Resuelve la siguiente integral:

$$\int (x+3)e^{-2x}\,dx$$

Solución:

a) $F(x) = \int (xe^{2x^2})\,dx = \left[\begin{array}{c} t = 2x^2 \\ dt = 4xdx \\ dx = \dfrac{dt}{4x} \end{array}\right] = \int (xe^t)\,\dfrac{dt}{4x} = \dfrac{1}{4}\int e^t\,dt = \dfrac{e^t}{4} + C = \dfrac{e^{2x^2}}{4} + C$

$F(0) = \dfrac{1}{4} + C = -1 \Longrightarrow C = -\dfrac{5}{4} \Longrightarrow F(x) = \dfrac{e^{2x^2} - 5}{4}$

b)

c) $\int \dfrac{2}{2+e^x}\,dx = \left[\begin{array}{c} t = e^x \\ dt = e^x dx \\ dx = \dfrac{dt}{e^x} = \dfrac{dt}{t} \end{array}\right] = \int \dfrac{2}{t(2+t)}\,dt = \left[\begin{array}{l} \dfrac{2}{t(2+t)} = \dfrac{A}{t} + \dfrac{B}{2+t} = \dfrac{A(2+t)+Bt}{t(2+t)} \\ 2 = A(2+t) + Bt \\ t = 0 \Longrightarrow A = 1 \\ t = -2 \Longrightarrow B = -1 \\ \dfrac{2}{t(2+t)} = \dfrac{1}{t} - \dfrac{1}{2+t} \end{array}\right] =$

$\int \dfrac{1}{t}\,dt - \int \dfrac{1}{2+t}\,dt = \ln|t| - \ln|2+t| + C = \ln|e^x| - \ln|2+e^x| + C = x - \ln|2+e^x| + C$

Problema 2 (2,5 puntos)

a) (1 punto)Calcular $\lim\limits_{x\to 2} \dfrac{\sqrt{x^3+x-1} - \sqrt{x^3+1}}{x-2}$.

b) (1,5 punto) Calcúlese $\int x(\ln x - 1)\,dx$

Solución:

a) $\lim\limits_{x\to 2} \dfrac{\sqrt{x^3+x-1} - \sqrt{x^3+1}}{x-2} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x\to 2} \dfrac{\frac{3x^2+1}{2\sqrt{x^3+x-1}} - \frac{3x^2}{2\sqrt{x^3+1}}}{1} = \dfrac{13}{6} - \dfrac{12}{6} = \dfrac{1}{6}$

b) $\int x(\ln x - 1)\,dx = \left[\begin{array}{c} u = \ln x - 1 \Longrightarrow du = \dfrac{1}{x}dx \\ dv = xdx \Longrightarrow v = \dfrac{x^2}{2} \end{array} \right] = \dfrac{x^2(\ln x - 1)}{2} - \dfrac{1}{2}\int x\,dx =$

$\dfrac{x^2(\ln x - 1)}{2} - \dfrac{x^2}{4} + C = \dfrac{2x^2(\ln x - 1) - x^2}{4} + C = \dfrac{2x^2 \ln x - 3x^2}{4} + C$

Problema 3 (2,5 puntos) Calcular:

a) (1,25 punto) $\lim\limits_{x\to 0} \dfrac{\sin x^2}{x^3 - 4x^2}$.

b) (1,25 punto) $\displaystyle\int_0^{\frac{\pi}{2}} \sin x \cos^3 x\,dx$.

Solución:

a) $\lim\limits_{x\to 0} \dfrac{\sin x^2}{x^3 + 4x^2} = \left[\dfrac{0}{0}\right] \stackrel{L'H}{=} \lim\limits_{x\to 0} \dfrac{2x\cos x^2}{3x^2 + 8x} = \left[\dfrac{0}{0}\right] \stackrel{L'H}{=} \lim\limits_{x\to 0} \dfrac{2\cos x^2 - 4x^2 \sin x^2}{6x + 8} = \dfrac{2}{8} = \dfrac{1}{4}$

b) $F(x) = \displaystyle\int \sin x \cos^3 x\,dx = \left[\begin{array}{c} t = \cos x \\ dt = -\sin x\,dx \\ dx = -\dfrac{dt}{\sin x} \end{array} \right] = -\int \sin x \cdot t^3 \cdot \dfrac{dt}{\sin x} = -\int t^3\,dt =$

$-\dfrac{t^4}{4} + C = -\dfrac{\cos^4 x}{4} + C$

$\displaystyle\int_0^{\frac{\pi}{2}} \sin x \cos^3 x\,dx = F\left(\dfrac{\pi}{2}\right) - F(0) = \dfrac{1}{4}$

Problema 4 (2,5 puntos) Determinar la primitiva $F(x)$ de la función $f(x) = (x+1)e^{x+1}$ que cumple $F(0) = -1$.

Solución:

$F(x) = \displaystyle\int (x+1)e^{x+1}\,dx \left[\begin{array}{c} u = x + 1 \Longrightarrow du = dx \\ dv = e^{x+1}\,dx \Longrightarrow v = e^{x+1} \end{array} \right] = (x+1)e^{x+1} - \int e^{x+1}\,dx =$

$(x+1)e^{x+1} - e^{x+1} + C = xe^{x+1} + C$

$F(0) = 0 + C = -1 \Longrightarrow C = -1 \Longrightarrow F(x) = xe^{x+1} - 1$