Examen de Matemáticas 2ºBachillerato(CN)
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Problema 1 Sea la matriz

$$A = \begin{pmatrix} m & 1 & -1 \\ 3 & 0 & m \\ 1 & m & -4 \end{pmatrix}$$

a) Calcular los valores de m para los que la matriz A es inversible.

b) Calcular A^{-1} para $m = 1$.

Solución:

a)

$$\begin{vmatrix} m & 1 & -1 \\ 3 & 0 & m \\ 1 & m & -4 \end{vmatrix} = -m^3 - 2m + 12 = 0 \Longrightarrow m = 2$$

Si $m = 2 \Longrightarrow |A| = 0 \Longrightarrow \nexists A^{-1}$.

Si $m \neq 2 \Longrightarrow |A| \neq 0 \Longrightarrow \exists A^{-1}$.

b) $A = \begin{pmatrix} 1 & 1 & -1 \\ 3 & 0 & 1 \\ 1 & m & 1 \end{pmatrix} \Longrightarrow A^{-1} = \begin{pmatrix} -1/9 & 1/3 & 1/9 \\ 13/9 & -1/3 & -4/9 \\ 1/3 & 0 & -1/3 \end{pmatrix}$

Problema 2 Resolver la ecuación matricial $AX + B = 2I - CX$. Donde

$$A = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}; \quad B = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}$$

Solución:

$$AX+B = 2I-CX \Longrightarrow AX+CX = 2I-B \Longrightarrow (A+C)X = 2I-B \Longrightarrow X = (A+C)^{-1}(2I-B)$$

$$(A+C)^{-1} = \left[\begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}\right]^{-1} = \begin{pmatrix} 0 & 2 \\ 3 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} -1/2 & 1/3 \\ 1/2 & 0 \end{pmatrix}$$

$$(2I - B) = 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -2 & -1 \end{pmatrix}$$

$$X = (A+C)^{-1}(2I-B) = \begin{pmatrix} -1/2 & 1/3 \\ 1/2 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} -2/3 & 1/6 \\ 0 & -1/2 \end{pmatrix}$$

Problema 3 Resolver utilizando las propiedades de los determinantes:

$$\begin{vmatrix} 1 & 1 & x & 0 \\ 1 & x & 0 & 1 \\ 0 & 1 & 1 & x \\ x & 0 & 1 & 1 \end{vmatrix}$$

Solución:

$$\begin{vmatrix} 1 & 1 & x & 0 \\ 1 & x & 0 & 1 \\ 0 & 1 & 1 & x \\ x & 0 & 1 & 1 \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1+\mathrm{F}_2+\mathrm{F}_3+\mathrm{F}_4 \\ \mathrm{F}_2 \\ \mathrm{F}_3 \\ \mathrm{F}_4 \end{bmatrix} = \begin{vmatrix} x+2 & x+2 & x+2 & x+2 \\ 1 & x & 0 & 1 \\ 0 & 1 & 1 & x \\ x & 0 & 1 & 1 \end{vmatrix} =$$

$$(x+2)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 0 & 1 \\ 0 & 1 & 1 & x \\ x & 0 & 1 & 1 \end{vmatrix} = \begin{bmatrix} \mathrm{C}_1 \\ \mathrm{C}_2-\mathrm{C}_1 \\ \mathrm{C}_3-\mathrm{C}_1 \\ \mathrm{C}_4-\mathrm{C}_1 \end{bmatrix} = (x+2)\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & x-1 & -1 & 0 \\ 0 & 1 & 1 & x \\ x & -x & 1-x & 1-x \end{vmatrix} =$$

$$(x+2)\begin{vmatrix} x-1 & -1 & 0 \\ 1 & 1 & x \\ -x & 1-x & 1-x \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1 \\ \mathrm{F}_2+\mathrm{F}_1 \\ \mathrm{F}_3+\mathrm{F}_1 \end{bmatrix} = (x+2)\begin{vmatrix} x-1 & -1 & 0 \\ x & 0 & x \\ -1 & -x & 1-x \end{vmatrix} =$$

$$x(x+2)\begin{vmatrix} x-1 & -1 & 0 \\ 1 & 0 & 1 \\ -1 & -x & 1-x \end{vmatrix} = \begin{bmatrix} \mathrm{C}_1 \\ \mathrm{C}_2 \\ \mathrm{C}_3-\mathrm{C}_1 \end{bmatrix} =$$

$$x(x+2)\begin{vmatrix} x-1 & -1 & 1-x \\ 1 & 0 & 0 \\ -1 & -x & 2-x \end{vmatrix} = -x(x+2)\begin{vmatrix} -1 & 1-x \\ -x & 2-x \end{vmatrix} = x(x+2)(x^2-2x+2)$$