# Examen de Matemáticas 2º de Bachillerato CN
**Febrero 2023**

**Problema 1** Calcule una primitiva $F(x)$ de la función

$$f(x) = \frac{x-3}{x^2-1}$$

**Solución:**

$$\int \frac{x-3}{x^2-1}\,dx = \left[\begin{array}{c} x^2-1=0 \Longrightarrow x=1,\ \ x=-1 \\ \dfrac{x-3}{x^2-1} = \dfrac{A}{x-1} + \dfrac{B}{x+1} = \dfrac{A(x+1)+B(x-1)}{(x-1)(x+1)} \\ x-3 = A(x+1)+B(x-1) \\ x=-1 \Longrightarrow -4=-2B \Longrightarrow B=2 \\ x=1 \Longrightarrow -2=2A \Longrightarrow A=-1 \\ \dfrac{x-3}{x^2-1} = \dfrac{-1}{x-1} + \dfrac{2}{x+1} \end{array}\right] =$$

$$-\int \frac{1}{x-1}\,dx + 2\int \frac{1}{x+1}\,dx = -\ln|x-1| + 2\ln|x+1| + C$$

**Problema 2** Se considera la función $f(x) = xe^{-x^2}$. Calcular el valor de las integrales indefinidas $\int f(x)\,dx$ e $\int xe^{-x}\,dx$

**Solución:**

$$\int f(x)\,dx = \int xe^{-x^2}\,dx = -\frac{1}{2}e^{-x^2} + C$$

$$\int xe^{-x}\,dx = \left[\begin{array}{l} u = x \Longrightarrow du = dx \\ dv = e^{-x}\,dx \Longrightarrow v = -e^{-x} \end{array}\right] = -xe^{-x} + \int e^{-x}\,dx =$$

$$-xe^{-x} - e^{-x} + C = -e^{-x}(x+1) + C$$

**Problema 3** Calcule la integral indefinida $\int x^2\cos x\,dx$ **Solución:**

$$\int x^2\cos x\,dx = \left[\begin{array}{l} u = x^2 \Longrightarrow du = 2xdx \\ dv = \cos xdx \Longrightarrow v = \sin x \end{array}\right] = x^2\sin x - 2\int x\sin x\,dx =$$

$\left[\begin{array}{l} u = x \Longrightarrow du = dx \\ dv = \sin xdx \Longrightarrow v = -\cos x \end{array}\right] = x^2\sin x - 2\left(-x\cos x + \int \cos x\,dx\right) = x^2\sin x + 2x\cos x - 2\sin x + C = (x^2-2)\sin x + 2x\cos x + C$

**Problema 4** Calcular una primitiva de la función $f(x) = x^2\ln x$, que se anule en $x = 1$.

**Solución:**

$$F(x)=\int x^2\ln x\,dx=\left[\begin{array}{l} u=\ln x\Longrightarrow du=\dfrac{1}{x}dx\\ dv=x^2dx\Longrightarrow v=\dfrac{x^3}{3}\end{array}\right]=\frac{x^3\ln x}{3}-\frac{1}{3}\int x^2\,dx=$$

$$\frac{x^3\ln x}{3}-\frac{x^3}{9}+C=\frac{3x^3\ln x-x^3}{9}+C=\frac{x^3(3\ln x-1)}{9}+C$$

$$F(1)=-\frac{1}{9}+C=0\Longrightarrow C=\frac{1}{9}\Longrightarrow F(x)=\frac{x^3(3\ln x-1)+1}{9}$$

**Problema 5** Se pide:

a) Calcule la integral indefinida $\displaystyle\int\frac{\sqrt{x}}{1+x}\,dx$

b) Determine la primitiva de $\dfrac{\sqrt{x}}{1+x}$ que pasa por el punto $(1,2)$.

c) Calcule el límite $\displaystyle\lim_{x\longrightarrow+\infty}\frac{\sqrt{x}}{1+x}$

**Solución:**

a) $$\int\frac{\sqrt{x}}{1+x}\,dx=\left[\begin{array}{l} t=\sqrt{x}\Longrightarrow x=t^2\\ dt=\dfrac{1}{2\sqrt{x}}dx\\ dx=2tdt\end{array}\right]=\int\frac{t}{1+t^2}\,2tdt=2\int\frac{t^2}{1+t^2}\,dt=$$

$$2\left(\int\left(1-\frac{1}{1+t^2}\right)\,dt\right)=2(t-\arctan t)+C=2(\sqrt{x}-\arctan\sqrt{x})+C$$

b) $F(x)=2(\sqrt{x}-\arctan\sqrt{x})+C\Longrightarrow F(1)=2(1-\arctan 1)+C=2\left(1-\dfrac{\pi}{4}\right)+C=$
$\dfrac{4-\pi}{2}+C=2\Longrightarrow C=\dfrac{\pi}{2}\Longrightarrow F(x)==2(\sqrt{x}-\arctan\sqrt{x})+\dfrac{\pi}{2}$

c) $$\lim_{x\longrightarrow+\infty}\frac{\sqrt{x}}{1+x}=\left[\frac{\infty}{\infty}\right]=\lim_{x\longrightarrow+\infty}\frac{\frac{1}{2\sqrt{x}}}{1}=0$$

**Problema 6** Calcula la derivada de las siguientes funciones y simplifica el resultado:

a) $f(x)=\ln\sqrt{\dfrac{1-\cos 2x}{\sin 2x}}$

b) $g(x)=\left(\dfrac{1}{x}\right)^{-x}$

**Solución:**

a) $f(x) = \ln\sqrt{\dfrac{1-\cos 2x}{\sin 2x}} = \dfrac{1}{2}\ln\left(\dfrac{1-\cos 2x}{\sin 2x}\right) = \dfrac{1}{2}\left[\ln(1-\cos 2x) - \ln\sin 2x\right]$

$f'(x) = \dfrac{1}{2}\left[\dfrac{2\sin 2x}{1-\cos 2x} - \dfrac{2\cos 2x}{\sin 2x}\right] = \dfrac{\sin 2x}{1-\cos 2x} - \cot 2x = \dfrac{2\sin x\cos x}{1-\cos^2 + \sin^2 x} - \cot 2x = \dfrac{2\sin x\cos x}{2\sin^2 x} - \cot 2x = \cot x - \cot 2x = \operatorname{cosec} 2x$

b) $\ln g(x) = -x\ln\left(\dfrac{1}{x}\right) = x\ln x$

$\dfrac{g'(x)}{g(x)} = 1 + \ln x \implies g'(x) = g(x)(1+\ln x) \implies g'(x) = \left(\dfrac{1}{x}\right)^{-x}(1+\ln x) = x^x(1+\ln x)$