Examen de Matemáticas 2ºBachillerato(CN)

Octubre 2017

Problema 1 Sea la matriz

$$A = \begin{pmatrix} 1 & m-1 & 3 \\ m & -1 & m \\ 1 & 4 & 7 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} 1 & m-1 & 3 \\ m & -1 & m \\ 1 & 4 & 7 \end{vmatrix} = -2(3m^2 - 7m + 2) = 0 \Longrightarrow m = 2, \;\; m = \frac{1}{3}$$

Si $m = 2$ o $m = -1/3 \Longrightarrow |A| = 0 \Longrightarrow$ no existe A^{-1}.

Si $m \neq 2$ y $m \neq 1/3 \Longrightarrow |A| \neq 0 \Longrightarrow$ existe A^{-1}.

2.

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 0 & -1 & 0 \\ 1 & 4 & 7 \end{pmatrix} \Longrightarrow A^{-1} = \begin{pmatrix} 7/4 & -19/4 & -3/4 \\ 0 & -1 & 0 \\ -1/4 & 5/4 & 1/4 \end{pmatrix}$$

Problema 2 Resolver la ecuación matricial $AX + B = C - X$. Donde

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}; \;\; B = \begin{pmatrix} -1 & 5 \\ 2 & 3 \end{pmatrix}; \;\; C = \begin{pmatrix} 3 & 4 \\ 4 & 0 \end{pmatrix}$$

Solución:

$$AX + B = C - X \Longrightarrow X = (A + I)^{-1}(C - B)$$

$$A+I = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}, \quad (A+I)^{-1} = \begin{pmatrix} 3/10 & -1/10 \\ -1/5 & 2/5 \end{pmatrix}$$

$$C - B = \begin{pmatrix} 3 & 4 \\ 4 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 5 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 2 & -3 \end{pmatrix}$$

$$X = (A + I)^{-1}(C - B) = \begin{pmatrix} 3/10 & -1/10 \\ -1/5 & 2/5 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Problema 3 Resolver utilizando las propiedades de los determinantes:

$$\begin{vmatrix} x & 1 & 1 & 0 \\ 0 & x & 1 & 1 \\ 1 & 0 & x & 1 \\ 1 & 1 & 0 & x \end{vmatrix}$$

Solución:

$$\begin{vmatrix} x & 1 & 1 & 0 \\ 0 & x & 1 & 1 \\ 1 & 0 & x & 1 \\ 1 & 1 & 0 & x \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1+\mathrm{F}_2+\mathrm{F}_3+\mathrm{F}_4 \\ \mathrm{F}_2 \\ \mathrm{F}_3 \\ \mathrm{F}_4 \end{bmatrix} = \begin{vmatrix} x+2 & x+2 & x+2 & x+2 \\ 0 & x & 1 & 1 \\ 1 & 0 & x & 1 \\ 1 & 1 & 0 & x \end{vmatrix} =$$

$$(x+2)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & x & 1 & 1 \\ 1 & 0 & x & 1 \\ 1 & 1 & 0 & x \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1 \\ \mathrm{F}_2 \\ \mathrm{F}_3-\mathrm{F}_1 \\ \mathrm{F}_4-\mathrm{F}_1 \end{bmatrix} = (x+2)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & x & 1 & 1 \\ 0 & -1 & x-1 & 0 \\ 0 & 0 & -1 & x-1 \end{vmatrix} =$$

$$(x+2)\begin{vmatrix} x & 1 & 1 \\ -1 & x-1 & 0 \\ 0 & -1 & x-1 \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1+\mathrm{F}_3 \\ \mathrm{F}_2 \\ \mathrm{F}_3 \end{bmatrix} = (x+2)\begin{vmatrix} x & 0 & x \\ -1 & x-1 & 0 \\ 0 & -1 & x-1 \end{vmatrix} =$$

$$x(x+2)\begin{vmatrix} 1 & 0 & 1 \\ -1 & x-1 & 0 \\ 0 & -1 & x-1 \end{vmatrix} = \begin{bmatrix} \mathrm{C}_1 \\ \mathrm{C}_2 \\ \mathrm{C}_3-\mathrm{C}_1 \end{bmatrix} = x(x+2)\begin{vmatrix} 1 & 0 & 0 \\ -1 & x-1 & 1 \\ 0 & -1 & x-1 \end{vmatrix} =$$

$$x(x+2)\begin{vmatrix} x-1 & 1 \\ -1 & x-1 \end{vmatrix} = x(x+2)(x^2-2x+2)$$