

Examen de Matemáticas 2º de Bachillerato
Enero 2013

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \rightarrow \infty} \left[\frac{\sqrt[3]{3 + 5x - 8x^3}}{1 + 2x} \right]$
2. $\lim_{x \rightarrow 0^+} \left(\frac{1}{x^2} \right)^{\tan x}$
3. $\lim_{x \rightarrow 0} \frac{\cos 2x - e^{-x} - x}{x \sin x}$
4. $\lim_{x \rightarrow 0} \frac{e^x - x \cos x - 1}{\sin x - x + 1 - \cos x}$
5. $\lim_{x \rightarrow \infty} \frac{2^x + x}{e^x}$
6. $\lim_{x \rightarrow \pi/2} \frac{\cos(\frac{\pi}{2} \sin x)}{(x - \frac{\pi}{2})}$
7. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - \sqrt{x^2 - x})$
8. $\lim_{x \rightarrow \infty} \ln(x^2 - 9), \quad \lim_{x \rightarrow 3^+} \ln(x^2 - 9)$

Solución:

1. $\lim_{x \rightarrow \infty} \left[\frac{\sqrt[3]{3 + 5x - 8x^3}}{1 + 2x} \right] = -1$
2. $\lim_{x \rightarrow \infty} \left(\frac{1}{x^2} \right)^{\tan x} = 1$
3. $\lim_{x \rightarrow 0} \frac{\cos 2x - e^{-x} - x}{x \sin x} = -\frac{5}{2}$
4. $\lim_{x \rightarrow 0} \frac{e^x - x \cos x - 1}{\sin x - x + 1 - \cos x} = 1$
5. $\lim_{x \rightarrow \infty} \frac{2^x + x}{e^x} = 0$
6. $\lim_{x \rightarrow \pi/2} \frac{\cos(\frac{\pi}{2} \sin x)}{(x - \frac{\pi}{2})} = \frac{\pi}{4}$
7. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - \sqrt{x^2 - x}) = \frac{1}{2}$

$$8. \lim_{x \rightarrow \infty} \ln(x^2 - 9) = \infty, \quad \lim_{x \rightarrow 3^+} \ln(x^2 - 9) = -\infty$$

Problema 2 Calcular las siguientes integrales:

$$1. \int \left(\frac{x^3 + 2\sqrt[3]{x^2} - x}{x^2} - 5e^x \right) dx$$

$$2. \int \left(\frac{x^2 - 4\sqrt[4]{x} - x}{x^2} - 2e^x \right) dx$$

$$3. \int x^2 e^{5x^3+8} dx$$

$$4. \int \frac{2x}{4x^2 - 1} dx$$

$$5. \int \frac{x^2}{1 + (4 - x^3)^2} dx$$

Solución:

$$1. \int \left(\frac{x^3 + 2\sqrt[3]{x^2} - x}{x^2} - 5e^x \right) dx = \frac{x^2}{2} - 6x^{-1/3} - \ln|x| - 5e^x + C$$

$$2. \int \left(\frac{x^2 - 4\sqrt[4]{x} - x}{x^2} - 2e^x \right) dx = x + \frac{16x^{-3/4}}{3} - \ln|x| - 2e^x + C$$

$$3. \int x^2 e^{5x^3+8} dx = \frac{1}{15} e^{5x^3+8} + C$$

$$4. \int \frac{2x}{4x^2 - 1} dx = \frac{\ln|4x^2 - 1|}{4} + C$$

$$5. \int \frac{x^2}{1 + (4 - x^3)^2} dx = -\frac{1}{3} \arctan(4 - x^3) + C$$