

**Examen de Matemáticas 2º Bachillerato (CN)**  
**Octubre 2012**

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**Problema 1** Sea la matriz

$$A = \begin{pmatrix} 3 & m & 0 \\ m & 2 & 2 \\ -m & 3 & 4 \end{pmatrix}$$

1. Calcular los valores de  $m$  para los que la matriz  $A$  es inversible.
2. Calcular  $A^{-1}$  para  $m = 0$ .

**Solución:**

1.

$$\begin{vmatrix} 3 & m & 0 \\ m & 2 & 2 \\ -m & 3 & 4 \end{vmatrix} = -6(m^2 - 1) = 0 \implies m = \pm 1$$

Si  $m = 1$  o  $m = -1 \implies |A| = 0 \implies$  no existe  $A^{-1}$ .

Si  $m \neq 1$  y  $m \neq -1 \implies |A| \neq 0 \implies$  existe  $A^{-1}$ .

2.

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 3 & 4 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -3/2 & 1 \end{pmatrix}$$

**Problema 2** Resolver la ecuación matricial  $AX - X + B = C$ . Donde

$$A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 4 \\ -1 & 3 \end{pmatrix}; \quad C = \begin{pmatrix} 3 & 7 \\ 1 & 0 \end{pmatrix}$$

**Solución:**

$$AX - X + B = C \implies X = (A - I)^{-1}(C - B)$$

$$A - I = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}, \quad (A - I)^{-1} = \begin{pmatrix} 1/2 & -1/2 \\ 0 & 1 \end{pmatrix}$$

$$C - B = \begin{pmatrix} 3 & 7 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 4 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 2 & -3 \end{pmatrix}$$

$$X = X = (A - I)^{-1}(C - B) = \begin{pmatrix} 1/2 & -1/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ 2 & -3 \end{pmatrix}$$

**Problema 3** Resolver, utilizando las propiedades de los determinantes, calcular:

$$\begin{vmatrix} 1 & x & 1 & x \\ x & 1 & x & 1 \\ 1 & 1 & x & x \\ x & x & 1 & 1 \end{vmatrix}$$

**Solución:**

$$\begin{aligned} \begin{vmatrix} 1 & x & 1 & x \\ x & 1 & x & 1 \\ 1 & 1 & x & x \\ x & x & 1 & 1 \end{vmatrix} &= \begin{bmatrix} F_1 + F_2 + F_3 + F_4 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{vmatrix} 2x+2 & 2x+2 & 2x+2 & 2x+2 \\ x & 1 & x & 1 \\ 1 & 1 & x & x \\ x & x & 1 & 1 \end{vmatrix} = \\ 2(x+1) \begin{vmatrix} 1 & 1 & 1 & 1 \\ x & 1 & x & 1 \\ 1 & 1 & x & x \\ x & x & 1 & 1 \end{vmatrix} &= \begin{bmatrix} C_1 \\ C_2 - C_1 \\ C_3 - C_1 \\ C_4 - C_1 \end{bmatrix} = 2(x+1) \begin{vmatrix} 1 & 0 & 0 & 0 \\ x & 1-x & 0 & 1-x \\ 1 & 0 & x-1 & x-1 \\ x & 0 & 1-x & 1-x \end{vmatrix} = \\ 2(x+1) \begin{vmatrix} 1-x & 0 & 1-x \\ 0 & x-1 & x-1 \\ 0 & 1-x & 1-x \end{vmatrix} &= -2(x-1)(x+1) \begin{vmatrix} x-1 & x-1 \\ 1-x & 1-x \end{vmatrix} = 0 \end{aligned}$$