

**Examen de Matemáticas 2º Bachillerato (CN)**  
**Octubre 2011**

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**Problema 1** Sea la matriz

$$A = \begin{pmatrix} -2 & m & 0 \\ m & 2 & -m \\ 1 & -3 & m \end{pmatrix}$$

1. Calcular los valores de  $m$  para los que la matriz  $A$  es inversible.
2. Calcular  $A^{-1}$  para  $m = -1$ .

**Solución:**

1.

$$\begin{vmatrix} -2 & m & 0 \\ m & 2 & -m \\ 1 & -3 & m \end{vmatrix} = -m^3 - m^2 + 2m = 0 \implies m = 0, m = 1, m = -2$$

Si  $m = 0$  o  $m = 1$  o  $m = -2 \implies |A| = 0 \implies$  no existe  $A^{-1}$ .

Si  $m = 0$  y  $m = 1$  y  $m = -2 \implies |A| \neq 0 \implies$  existe  $A^{-1}$ .

2.

$$A = \begin{pmatrix} -2 & -1 & 0 \\ -1 & 2 & 1 \\ 1 & -3 & -1 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} -1/2 & 1/2 & 1/2 \\ 0 & -1 & -1 \\ -1/2 & 7/2 & 5/2 \end{pmatrix}$$

**Problema 2** Resolver la ecuación matricial  $AX - X = B - CX$ . Donde

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}; B = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}; C = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$$

**Solución:**

$$AX - X = B - CX \implies (A + C - I)X = B \implies X = (A + C - I)^{-1}B$$

$$(A + C - I)^{-1} = \begin{pmatrix} 4/3 & -1 \\ -1 & 1 \end{pmatrix}$$

$$X = (A + C - I)^{-1}B = \begin{pmatrix} -7/3 & 11/3 \\ 2 & -3 \end{pmatrix}$$

**Problema 3** Resolver, utilizando las propiedades de los determinantes, calcular:

$$\begin{vmatrix} x & 1 & 1 & x \\ 1 & x & x & 1 \\ x & 1 & x & 1 \\ 1 & x & 1 & x \end{vmatrix}$$

**Solución:**

$$\begin{aligned} \begin{vmatrix} x & 1 & 1 & x \\ 1 & x & x & 1 \\ x & 1 & x & 1 \\ 1 & x & 1 & x \end{vmatrix} &= \begin{bmatrix} F_1 + F_2 + F_3 + F_4 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{vmatrix} 2x+2 & 2x+2 & 2x+2 & 2x+2 \\ 1 & x & x & 1 \\ x & 1 & x & 1 \\ 1 & x & 1 & x \end{vmatrix} = \\ 2(x+1) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & x & 1 \\ x & 1 & x & 1 \\ 1 & x & 1 & x \end{vmatrix} &= \begin{bmatrix} C_1 \\ C_2 - C_1 \\ C_3 - C_1 \\ C_4 - C_1 \end{bmatrix} = 2(x+1) \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & x-1 & x-1 & 0 \\ x & 1-x & 0 & 1-x \\ 1 & x-1 & 0 & x-1 \end{vmatrix} = \\ 2(x+1) \begin{vmatrix} x-1 & x-1 & 0 \\ 1-x & 0 & x-1 \\ x-1 & 0 & x-1 \end{vmatrix} &= 2(x-1)(x+1) \begin{vmatrix} 1-x & 1-x \\ x-1 & x-1 \end{vmatrix} = 0 \end{aligned}$$