Examen de Matemáticas 1º de Bachillerato CN

Marzo 2023

Problema 1 Calcular las siguientes integrales:

a) $\displaystyle\int \frac{3x^2}{5-7x^3}\,dx$

b) $\displaystyle\int 7x(2x^2+1)^{19}\,dx$

c) $\displaystyle\int \frac{5x^2\cos x-8x^2e^x-6x+3}{x^2}\,dx$

d) $\displaystyle\int \frac{9x^3-3\sqrt[5]{x^2}-x}{x^2}\,dx$

e) $\displaystyle\int \frac{3x^3\sin(x^2-1)+x^3e^{7x^2-2}-5x+1}{x^2}\,dx$

f) $\displaystyle\int \frac{-9}{1+x^2}\,dx$

Solución:

a) $\displaystyle\int \frac{3x^2}{5-7x^3}\,dx=-\frac{1}{7}\ln|5-7x^3|+C$

b) $\displaystyle\int 7x(2x^2+1)^{19}\,dx=\frac{7(2x^2+1)^{20}}{80}+C$

c) $\displaystyle\int \frac{5x^2\cos x-8x^2e^x-6x+3}{x^2}\,dx=5\sin x-8e^x-6\ln|x|-\frac{3}{x}+C$

d) $\displaystyle\int \frac{9x^3-3\sqrt[5]{x^2}-x}{x^2}\,dx=\frac{9x^2}{2}+\frac{5}{x^{3/5}}-\ln|x|+C$

e) $\displaystyle\int \frac{3x^3\sin(x^2-1)+x^3e^{7x^2-2}-5x+1}{x^2}\,dx=-\frac{3\cos(x^2-1)}{2}-\frac{e^{7x^2-2}}{14}-\frac{1}{x}-5\ln|x|+$ C

f) $\displaystyle\int \frac{-9}{1+x^2}\,dx=-9\arctan x+C$

Problema 2 Calcular la primera derivada de las siguientes funciones:

a) $y=\ln\sqrt[7]{\dfrac{x^3\cos(x^2+1)}{e^{x^2-3}\sin x}}$

b) $y = (\sin x)^{x^2+1}$

c) $y = \dfrac{\arctan(x^4-5)(3x+1)}{x^2+1}$

d) $y = \csc(5x-1)^2 \sec^2(x^2-7)$

e) $y = 4^{\cos^2 x - \sin x} \log_3(3x^2 + \cos x)$

f) $y = (\sqrt{x^2+8})^{\arctan x}$

Solución:

a) $y = \ln \sqrt[7]{\dfrac{x^3\cos(x^2+1)}{e^{x^2-3}\sin x}} = \dfrac{1}{7}\left(3\ln x + \ln\cos(x^2+1) - (x^2-3)\ln e - \ln(\sin x)\right) \Longrightarrow$

$$y' = \frac{1}{7}\left(\frac{3}{x} + \frac{-2x\sin(x^2+1)}{\cos(x^2+1)} - 2x - \frac{\cos x}{\sin x}\right)$$

b) $y = (\sin x)^{x^2+1} \Longrightarrow y' = (\sin x)^{x^2+1}\left(2x\ln\sin x + (x^2+1)\dfrac{\cos x}{\sin x}\right)$

c) $y = \dfrac{\arctan(x^4-5)(3x+1)}{x^2+1} \Longrightarrow$
$y' = \dfrac{\left(\frac{4x^3}{1+(x^4-5)^2}(3x+1) + 3\arctan(x^4-5)\right)(x^2+1) - \arctan(x^4-5)(3x+1)2x}{(x^2+1)^2}$

d) $y = \csc(5x-1)^2\sec^2(x^2-7) \Longrightarrow y' = -10(5x-1)\csc(5x-1)^2\cot(5x-1)^2\sec^2(x^2-7) + \csc(5x-1)^2 2\sec(x^2-7)2x\sec(x^2-7)\tan(x^2-7)$

e) $y = 4^{\cos^2 x-\sin x}\log_3(3x^2+\cos x) \Longrightarrow y' = (2\cos x(-\sin x)-\cos x)4^{\cos^2 x-\sin x}\log_3(3x^2+\cos x) + 4^{\cos^2 x-\sin x}\dfrac{6x-\sin x}{(3x^2+\cos x)\ln 3}$

f) $y = (\sqrt{x^2+8})^{\arctan x} \Longrightarrow y' = (\sqrt{x^2+8})^{\arctan x}\left(\dfrac{1}{1+x^2}\ln\sqrt{x^2+8} + \arctan x\dfrac{\frac{2x}{2\sqrt{x^2+8}}}{\sqrt{x^2+8}}\right)$

Problema 3 Calcular los siguientes límites:

a) $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{5x^2+6x-1} - \sqrt{5x^2+3}\right)$

b) $\lim\limits_{x\longrightarrow 1}\dfrac{x^4-8x^3+9x^2-7x+5}{2x^3+6x^2-11x+3}$

c) $\lim\limits_{x\longrightarrow 3}\dfrac{\sqrt{x^2-2}-\sqrt{2x+1}}{x-3}$

d) $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{4x^2-3x-1}{4x^2-5}\right)^{7x}$

e) $\lim_{x \longrightarrow \infty} \frac{e^{6x^2-2x+7}}{3x+1}$

f) $\lim_{x \longrightarrow \infty} \frac{e^{5x-7}-3}{e^{x+1}-1}$

g) $\lim_{x \longrightarrow 0} \frac{\sin^2 x - 5x}{2x \cos x}$

Solución:

a) $\lim_{x \longrightarrow \infty} \left(\sqrt{5x^2+6x-1} - \sqrt{5x^2+3} \right) = \frac{3\sqrt{5}}{5}$

b) $\lim_{x \longrightarrow 1} \frac{x^4-8x^3+9x^2-7x+5}{2x^3+6x^2-11x+3} = -\frac{9}{7}$

c) $\lim_{x \longrightarrow 3} \frac{\sqrt{x^2-2}-\sqrt{2x+1}}{x-3} = \frac{2\sqrt{7}}{7}$

d) $\lim_{x \longrightarrow \infty} \left(\frac{4x^2-3x-1}{4x^2-5} \right)^{7x} = e^{-21/4}$

e) $\lim_{x \longrightarrow \infty} \frac{e^{6x^2-2x+7}}{3x+1} = \infty$

f) $\lim_{x \longrightarrow \infty} \frac{e^{5x-7}-3}{e^{5x+1}-1} = e^{-8}$

g) $\lim_{x \longrightarrow 0} \frac{\sin^2 x - 5x}{2x \cos x} = -\frac{5}{2}$

Problema 4 Calcular las rectas tangente y normal de las siguientes funciones:

a) $f(x) = \frac{2x-1}{x+1}$ en el punto $x = 2$.

b) $f(x) = (x+2)e^{x-1}$ en el punto $x = 1$.

Solución:

a) $b = f(a) \Longrightarrow b = f(2) = 1$ e $y - b = m(x-a)$

$$f'(x) = -\frac{3}{(x+1)^2} \Longrightarrow m = f'(2) = \frac{1}{3}$$

Recta Tangente: $y - 1 = \frac{1}{3}(x-2)$

Recta Normal: $y - 1 = -3(x-2)$

b) $b = f(a) \Longrightarrow b = f(1) = 3$ e $y - b = m(x - a)$

$$f'(x) = (x + 3)e^{x-1} \Longrightarrow m = f'(1) = 4$$

Recta Tangente: $y - 3 = 4(x - 1)$

Recta Normal: $y - 3 = -\dfrac{1}{4}(x - 1)$