Examen de Matemáticas 1º de Bachillerato CN

Marzo 2020

Problema 1 Calcular las siguientes integrales:

1. $\displaystyle\int \frac{7x^2}{1+2x^3}\,dx$

2. $\displaystyle\int 5x(3x^2+1)^{22}\,dx$

3. $\displaystyle\int \frac{5x^2\cos x - 3x^2e^x + 5x - 1}{x^2}\,dx$

4. $\displaystyle\int \frac{3x^3 - 2\sqrt[5]{x^2} + 8x}{x^2}\,dx$

5. $\displaystyle\int \frac{7x^3\sin(x^2+1) + 2x^3e^{5x^2+9} - 3x + 2}{x^2}\,dx$

6. $\displaystyle\int \frac{9}{1+x^2}\,dx$

Solución:

1. $\displaystyle\int \frac{7x^2}{1+2x^3}\,dx = \frac{7}{6}\ln|1+2x^3| + C$

2. $\displaystyle\int 5x(3x^2+1)^{22}\,dx = \frac{5(3x^2+1)^{23}}{138} + C$

3. $\displaystyle\int \frac{5x^2\cos x - 3x^2e^x + 5x - 1}{x^2}\,dx = 5\sin x - 3e^x + 5\ln|x| + \frac{1}{x} + C$

4. $\displaystyle\int \frac{3x^3 - 2\sqrt[5]{x^2} + 8x}{x^2}\,dx = \frac{3x^2}{2} + \frac{10x^{-3/5}}{3} + 8\ln|x| + C$

5. $\displaystyle\int \frac{7x^3\sin(x^2+1) + 2x^3e^{5x^2+9} - 3x + 2}{x^2}\,dx = -\frac{7\cos(x^2+1)}{2} + \frac{e^{5x^2+9}}{5} - \frac{2}{x} - 3\ln|x| + C$

6. $\displaystyle\int \frac{9}{1+x^2}\,dx = 9\arctan x + C$

Problema 2 Calcular la primera derivada de las siguientes funciones:

1. $y = \ln\sqrt[3]{\dfrac{x^4\cos(x^2-7)}{e^{x^2+3}\sin x}}$

2. $y = (\sin x)^{x^3-1}$

3. $y = \dfrac{\arctan(x^4+2)(5x-1)}{x^2+3}$

4. $y = \csc(2x+1)^2 \sec^2(x^2-3)$

5. $y = 9^{\cos^2 x - \sin x} \log_3(2x^2 - \cos x)$

6. $y = (\sqrt{x^2-3})^{\arctan x}$

Solución:

1. $y = \ln \sqrt[3]{\dfrac{x^4 \cos(x^2-7)}{e^{x^2+3} \sin x}} = \frac{1}{3}\left(4\ln x + \ln\cos(x^3-7) - (x^2+3)\ln e - \ln(\sin x)\right) \Longrightarrow$

$$y' = \frac{1}{3}\left(\frac{4}{x} + \frac{-3x^2\sin(x^3-7)}{\cos(x^3-7)} - 2x - \frac{\cos x}{\sin x}\right)$$

2. $y = (\sin x)^{x^3-1} \Longrightarrow y' = (\sin x)^{x^3-1}\left(3x^2\ln\sin x + (x^5-1)\frac{\cos x}{\sin x}\right)$

3. $y = \dfrac{\arctan(x^4+2)(5x-1)}{x^2+3} \Longrightarrow$
$y' = \frac{\left(\frac{4x^3}{1+(x^4+2)^2}(5x-1)+5\arctan(x^4+2)\right)(x^2+3)-\arctan(x^4+2)(5x-1)2x}{(x^2+3)^2}$

4. $y = \csc(2x+1)^2\sec^2(x^2-3) \Longrightarrow y' = -2(2x+1)\csc(2x+1)^2\cot(2x+1)^2\sec^2(x^2-3) + \csc(2x+1)^2 2\sec(x^2-3)2x\sec(x^2-3)\tan(x^2-3)$

5. $y = 9^{\cos^2 x - \sin x}\log_3(2x^2-\cos x) \Longrightarrow y' = (2\cos x(-\sin x) - \cos x)9^{\cos^2 x - \sin x}\log_3(2x^2-\cos x) + 9^{\cos^2 x - \sin x}\frac{4x+\sin x}{(3x^2-\cos x)\ln 3}$

6. $y = (\sqrt{x^2-3})^{\arctan x} \Longrightarrow y' = (\sqrt{x^2-3})^{\arctan x}\left(\frac{1}{1+x^2}\ln\sqrt{x^2-3} + \arctan x\frac{\frac{2x}{2\sqrt{x^2-3}}}{\sqrt{x^2-3}}\right)$

Problema 3 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{4x^2-7x+1} - \sqrt{4x^2+9x-2}\right)$

2. $\lim\limits_{x\longrightarrow 1}\dfrac{8x^4-5x^2-4x+1}{6x^5-9x+3}$

3. $\lim\limits_{x\longrightarrow 7}\dfrac{\sqrt{x^2-8}-\sqrt{6x-1}}{x-7}$

4. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{5x^2-2x+3}{5x^2-7}\right)^{2x}$

5. $\displaystyle\lim_{x\longrightarrow\infty} \frac{e^{3x^2+9}}{5x-1}$

6. $\displaystyle\lim_{x\longrightarrow\infty} \frac{e^{2x+1}+8}{e^{3x-2}-3}$

7. $\displaystyle\lim_{x\longrightarrow 0} \frac{\sin^2 x-4x}{2x\cos x}$

Solución:

1. $\displaystyle\lim_{x\longrightarrow\infty} \left(\sqrt{4x^2-7x+1}-\sqrt{4x^2+9x-2}\right)=-4$

2. $\displaystyle\lim_{x\longrightarrow 1} \frac{8x^4-5x^2-4x+1}{6x^5-9x+3}=\frac{6}{7}$

3. $\displaystyle\lim_{x\longrightarrow 7} \frac{\sqrt{x^2-8}-\sqrt{6x-1}}{x-7}=\frac{4\sqrt{41}}{41}$

4. $\displaystyle\lim_{x\longrightarrow\infty} \left(\frac{5x^2-2x+3}{5x^2-7}\right)^{2x}=e^{-4/5}$

5. $\displaystyle\lim_{x\longrightarrow\infty} \frac{e^{3x^2+9}}{5x-1}=\infty$

6. $\displaystyle\lim_{x\longrightarrow\infty} \frac{e^{2x+1}+8}{e^{3x-2}-3}=0$

7. $\displaystyle\lim_{x\longrightarrow 0} \frac{\sin^2 x-4x}{2x\cos x}=-2$

Problema 4 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x)=\dfrac{4x+3}{x-1}$ en el punto $x=2$.

2. $f(x)=(x+7)e^{x+1}$ en el punto $x=-1$.

Solución:

1. $b=f(a)\Longrightarrow b=f(2)=11$ e $y-b=m(x-a)$

$$f'(x)=-\frac{7}{(x-1)^2}\Longrightarrow m=f'(2)=-7$$

Recta Tangente: $y-11=-7(x-2)$

Recta Normal: $y-11=\frac{1}{7}(x-2)$

2. $b = f(a) \Longrightarrow b = f(-1) = 6$ e $y - b = m(x - a)$

$$f'(x) = (x+8)e^{x+1} \Longrightarrow m = f'(-1) = 7$$

Recta Tangente: $y - 6 = 7(x+1)$

Recta Normal: $y - 6 = -\frac{1}{7}(x+1)$