Examen de Matemáticas 1º de Bachillerato CN

Diciembre 2018

Problema 1 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty}\dfrac{5x^4+2x^2-3x+1}{7x^4+3x+1}$

2. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{2x^2-7x+8}{5x^2-x+1}\right)^{7x^2+2}$

3. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{5x^2-2x+1}{5x^2-3}\right)^{7x}$

4. $\lim\limits_{x\longrightarrow\infty}\dfrac{\sqrt{5x^4-2x^3+3x^2-x+1}}{5x^2+8x-5}$

5. $\lim\limits_{x\longrightarrow 1}\dfrac{5x^4+3x^3-11x^3+2x+1}{2x^3-4x^2+x+1}$

6. $\lim\limits_{x\longrightarrow 2}\dfrac{3x^3-5x^2+2x-8}{2x^3-3x^2+x-6}$

7. $\lim\limits_{x\longrightarrow 7}\dfrac{\sqrt{x^2-2}-\sqrt{6x+5}}{x-7}$

8. $\lim\limits_{x\longrightarrow 5}\dfrac{\sqrt{x^2-6}-\sqrt{3x+4}}{x-5}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty}\dfrac{5x^4+2x^2-3x+1}{7x^4+3x+1}=\dfrac{5}{7}$

2. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{2x^2-7x+8}{5x^2-x+1}\right)^{7x^2+2}=0$

3. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{5x^2-2x+1}{5x^2-3}\right)^{7x}=e^{-14/5}$

4. $\lim\limits_{x\longrightarrow\infty}\dfrac{\sqrt{5x^4-2x^3+3x^2-x+1}}{5x^2+8x-5}=\dfrac{\sqrt{5}}{5}$

5. $\lim\limits_{x\longrightarrow 1}\dfrac{5x^4+3x^3-11x^3+2x+1}{2x^3-4x^2+x+1}=2$

6. $\lim\limits_{x\longrightarrow 2}\dfrac{3x^3-5x^2+2x-8}{2x^3-3x^2+x-6}=\dfrac{18}{13}$

7. $\lim\limits_{x \longrightarrow 7} \dfrac{\sqrt{x^2-2}-\sqrt{6x+5}}{x-7} = \dfrac{4\sqrt{47}}{47}$

8. $\lim\limits_{x \longrightarrow 5} \dfrac{\sqrt{x^2-6}-\sqrt{3x+4}}{x-5} = \dfrac{7\sqrt{19}}{38}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{2x^3-3x^2-x-8}$
2. $y = \ln(7x^3+8)$
3. $y = (x^2+8x+1)^{19}$
4. $y = (2x^2-x+1)(x^3+3x^2-7)$
5. $y = \dfrac{5x^2+8}{7x-2}$
6. $y = \ln \frac{x^2-3x-9}{x^2+5x-1}$
7. $y = (x^2+9)^{\cos x}$
8. $y = \arctan(x^2+7x-8)$
9. $y = \sqrt{3x^2+6x-2}$

Solución:

1. $y = e^{2x^3-3x^2-x-8} \Longrightarrow y' = (6x^2-6x-1)e^{2x^3-3x^2-x-8}$
2. $y = \ln(7x^3+8) \Longrightarrow y' = \dfrac{21x^2}{7x^3+8}$
3. $y = (x^2+8x+1)^{19} \Longrightarrow y' = 19(x^2+8x+1)^{18}(2x+8)$
4. $y = (2x^2-x+1)(x^3+3x^2-7) \Longrightarrow y' = (4x-1)(x^3+3x^2-7) + (2x^2-x+1)(3x^2+6x)$
5. $y = \dfrac{5x^2+8}{7x-2} \Longrightarrow y' = \dfrac{(10x)(7x-2)-(5x^2+8)7}{(7x-2)^2}$
6. $y = \ln \frac{x^2-3x-9}{x^2+5x-1} = \ln(x^2-3x-9) - \ln(x^2+5x-1) \Longrightarrow y' = \dfrac{2x-3}{x^2-3x-9} - \dfrac{2x+5}{x^2+5x-1}$
7. $y = (x^2+9)^{\cos x} \Longrightarrow y' = (x^2+9)^{\cos x}\left(-\sin x \ln(x^2+9) + \cos x \frac{2x}{x^2+9}\right)$
8. $y = \arctan(x^2+7x-8) \Longrightarrow y' = \dfrac{2x+7}{1+(x^2+7x-8)^2}$

9. $y = \sqrt{3x^2 + 6x - 2} \Longrightarrow y' = \dfrac{6x + 6}{2\sqrt{3x^2 + 6x - 2}}$

Problema 3 Calcular las rectas tangente y normal a la siguiente funciones en el punto $x = 1$:

1. $f(x) = \dfrac{x^2 + 5x - 2}{x^2 + 1}$.

2. $f(x) = (x + 2)e^{x-1}$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = 2$ e $y - b = m(x - a)$

$$f'(x) = -\frac{5x^2 - 6x - 5}{(x^2 + 1)^2} \Longrightarrow m = f'(1) = \frac{3}{2}$$

   Recta Tangente: $y - 2 = \frac{3}{2}(x - 1)$

   Recta Normal: $y - 2 = -\frac{2}{3}(x - 1)$

2. $b = f(a) \Longrightarrow b = f(1) = 3$ e $y - b = m(x - a)$

$$f'(x) = (x + 3)e^{x-1} \Longrightarrow m = f'(1) = 4$$

   Recta Tangente: $y - 3 = 4(x - 1)$

   Recta Normal: $y - 2 = -\dfrac{1}{4}(x - 1)$