Examen de Matemáticas 1º de Bachillerato CN

Marzo 2018

Problema 1 Calcular los siguientes límites:

1. $\lim_{x\longrightarrow\infty}\left(\sqrt{x^2+x-1}-\sqrt{x^2+2x-3}\right)$
2. $\lim_{x\longrightarrow 7}\frac{\sqrt{x^2+2}-\sqrt{6x+9}}{x-7}$
3. $\lim_{x\longrightarrow 1}\frac{5x^3+6x^2-12x+1}{7x^3+x^2-10x+2}$
4. $\lim_{x\longrightarrow 0}\frac{x\cos x-e^x+1}{x\sin x}$
5. $\lim_{x\longrightarrow\infty}\frac{e^{5x}+2x}{e^{2x}+3x}$
6. $\lim_{x\longrightarrow 0}\frac{\sin x+xe^x}{\cos x-e^x}$
7. $\lim_{x\longrightarrow 0}\frac{\ln(x^2+1)}{\ln(5x^2+1)}$

Solución:

1. $\lim_{x\longrightarrow\infty}\left(\sqrt{x^2+x-1}-\sqrt{x^2+2x-3}\right)=-\frac{1}{2}$
2. $\lim_{x\longrightarrow 7}\frac{\sqrt{x^2+2}-\sqrt{6x+9}}{x-7}=\frac{4\sqrt{51}}{51}$
3. $\lim_{x\longrightarrow 1}\frac{5x^3+6x^2-12x+1}{7x^3+x^2-10x+2}=\frac{15}{13}$
4. $\lim_{x\longrightarrow 0}\frac{x\cos x-e^x+1}{x\sin x}=-\frac{1}{2}$
5. $\lim_{x\longrightarrow\infty}\frac{e^{5x}+2x}{e^{2x}+3x}=\infty$
6. $\lim_{x\longrightarrow 0}\frac{\sin x+xe^x}{\cos x-e^x}=-2$
7. $\lim_{x\longrightarrow 0}\frac{\ln(x^2+1)}{\ln(5x^2+1)}=\frac{1}{5}$

Problema 2 Calcular la primera derivada de las siguientes funciones:

1. $y = \ln \sqrt[7]{\dfrac{x^2\cos^3(3x)}{e^{7x}\sin x^2}}$

2. $y = (4x^2+8)^{\sin(3x)}$

3. $y = (\arcsin x)^{5x^2-1}$

4. $y = \log_3 \dfrac{9x^2+3}{\sqrt{x^2-1}}$

5. $y = \sqrt[8]{\dfrac{5x^2+1}{\cos^2(7x)}}$

6. $y = \csc^2(x^2+5)\log_5(x^2+1)$

7. $y = 7^{\arctan(x^2-1}\tan^2(x+3)$

Solución:

1. $y = \ln \sqrt[7]{\dfrac{x^2\cos^3(3x)}{e^{7x}\sin x^2}} = \frac{1}{7}\left(2\ln x + 3\ln\cos(3x) - (7x)\ln e - \ln(\sin x^2)\right) \Longrightarrow$

$$y' = \frac{1}{7}\left(\frac{2}{x} + 3\frac{-3\sin(3x)}{\cos(3x)} - \frac{7}{7x} - \frac{2x\cos x^2}{\sin x^2}\right)$$

2. $y = (4x^2+8)^{\sin(3x)} \Longrightarrow y' = (4x^2+8)^{\sin(3x)}\left(3\cos(3x)\ln(4x^2+8) + \sin(3x)\frac{8x}{4x^2+8}\right)$

3. $y = (\arcsin x)^{5x^2-1} \Longrightarrow y' = (\arcsin x)^{5x^2-1}\left(10x\ln(\arcsin x) + (5x^2-1)\frac{\frac{1}{\sqrt{1-x^2}}}{\arcsin x}\right)$

4. $y = \log_3 \dfrac{9x^2+3}{\sqrt{x^2-1}} = \log_3(9x^2+3) - \dfrac{1}{2}\log_3(x^2-1) \Longrightarrow y' = \dfrac{18x}{(9x^2+3)\ln 3} - \dfrac{1}{2}\dfrac{2x}{(x^2-1)\ln 3}$

5. $y = \sqrt[8]{\dfrac{5x^2+1}{\cos^2(7x)}} \Longrightarrow y' = \dfrac{1}{8}\left(\dfrac{5x^2+1}{\cos^2(7x)}\right)^{-7/2}\left(\dfrac{10x\cos^2(7x) - (5x^2+1)(2\cos(7x)(-7\sin(7x))}{\cos^4(7x)}\right)$

6. $y = \csc^2(x^2+5)\log_5(x^2+1) \Longrightarrow y' = -4x\csc^2(x^2+5)\cot(x^2+5)\log_5(x^2+1) + \csc^2(x^2+5)\dfrac{2x}{(x^2+1)\ln 5}$

7. $y = 7^{\arctan(x^2-1}\tan^2(x+3) \Longrightarrow y' = \dfrac{2x}{1+(x^2-1)^2}7^{\arctan(x^2-1}\ln 7\tan^2(x+3) + 7^{\arctan(x^2-1)}2\tan(x+3)\dfrac{1}{\cos^2(x+3)}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{x^2+5}{5x-3}$ en el punto $x=0$.
2. $f(x) = (x^2-1)e^{2x}$ en el punto $x=0$.

Solución:

1. $b = f(a) \Longrightarrow b = f(0) = -5/3$ e $y - b = m(x-a)$

   $$f'(x) = \frac{5x^2-6x-25}{(5x-3)^2} \Longrightarrow m = f'(0) = -\frac{25}{9}$$

   Recta Tangente: $y + \frac{5}{3} = -\frac{25}{9}x$

   Recta Normal: $y + \frac{5}{3} = \frac{9}{25}x$

2. $b = f(a) \Longrightarrow b = f(0) = -1$ e $y - b = m(x-a)$

   $$f'(x) = (2x^2+2x-2)e^{2x} \Longrightarrow m = f'(0) = -2$$

   Recta Tangente: $y + 1 = -2x$

   Recta Normal: $y + 1 = \dfrac{1}{2}x$

Problema 4 Calcular las siguientes integrales:

1. $\displaystyle\int 8xe^{3x^2+5}\,dx$
2. $\displaystyle\int \frac{7x}{5x^2+3}\,dx$
3. $\displaystyle\int 3x^2\cos(8x^3+2)\,dx$
4. $\displaystyle\int \frac{3x}{1+x^4}\,dx$
5. $\displaystyle\int \frac{3x^2+3x^2\cos x - x^2e^x + x}{x^2}\,dx$
6. $\displaystyle\int \frac{x^5-3x^4+2\sqrt[7]{x^3}-5x}{x^2}\,dx$

Solución:

1. $\displaystyle\int 8xe^{3x^2+5}\,dx = \frac{4}{3}e^{3x^2+5} + C$

2. $\displaystyle\int \frac{7x}{5x^2+3}\,dx = \frac{7}{10}\ln|5x^2+3| + C$

3. $\displaystyle\int 3x^2\cos(8x^3+2)\,dx = \frac{1}{8}\sin(8x^3+2) + C$

4. $\displaystyle\int \frac{3x}{1+x^4}\,dx = \frac{3}{2}\arctan x^2 + C$

5. $\displaystyle\int \frac{3x^2+3x^2\cos x - x^2e^x + x}{x^2}\,dx = 3x + 3\sin x - e^x + \ln|x| + C$

6. $\displaystyle\int \frac{x^5-3x^4+2\sqrt[7]{x^3}-5x}{x^2}\,dx = \frac{x^4}{4} - x^3 - \frac{7x^{-4/7}}{2} - 5\ln|x| + C$