Examen de Matemáticas 1º de Bachillerato CN

Diciembre 2017

Problema 1 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty}\dfrac{2x^4-7x^2-x-3}{3x^4-x+1}$

2. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{7x^2+5x+8}{3x^2-5}\right)^{x^2+7}$

3. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{2x^2-3x+1}{2x^2-1}\right)^{3x}$

4. $\lim\limits_{x\longrightarrow\infty}\dfrac{\sqrt{3x^4-x^3+5x^2-3x+1}}{5x^2-x-5}$

5. $\lim\limits_{x\longrightarrow 1}\dfrac{7x^4+2x^3-9x^3+3x-3}{x^3-5x^2+5x-1}$

6. $\lim\limits_{x\longrightarrow 2}\dfrac{2x^3-9x^2+5x+10}{x^3-3x^2+4x-4}$

7. $\lim\limits_{x\longrightarrow 7}\dfrac{\sqrt{x^2+2}-\sqrt{8x-5}}{x-7}$

8. $\lim\limits_{x\longrightarrow 5}\dfrac{\sqrt{x^2+4}-\sqrt{6x-1}}{x-5}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty}\dfrac{2x^4-7x^2-x-3}{3x^5-x+1}=\dfrac{2}{3}$

2. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{7x^2+5x+8}{3x^2-5}\right)^{x^2+7}=\infty$

3. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{2x^2-3x+1}{2x^2-1}\right)^{3x}=e^{-9/2}$

4. $\lim\limits_{x\longrightarrow\infty}\dfrac{\sqrt{3x^4-x^3+5x^2-3x+1}}{5x^2-x-5}=\dfrac{\sqrt{3}}{5}$

5. $\lim\limits_{x\longrightarrow 1}\dfrac{7x^4+2x^3-9x^3+3x-3}{x^3-5x^2+5x-1}=-5$

6. $\lim\limits_{x\longrightarrow 2}\dfrac{2x^3-9x^2+5x+10}{x^3-3x^2+4x-4}=-\dfrac{7}{4}$

7. $\lim\limits_{x\longrightarrow 7} \dfrac{\sqrt{x^2+2}-\sqrt{8x-5}}{x-7} = \dfrac{\sqrt{51}}{17}$

8. $\lim\limits_{x\longrightarrow 5} \dfrac{\sqrt{x^2+4}-\sqrt{6x-1}}{x-5} = \dfrac{2\sqrt{29}}{29}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{3x^3-5x^2-2x-2}$
2. $y = \ln(5x^3+11)$
3. $y = (x^2-3x+1)^{17}$
4. $y = (x^2-3x+1)(2x^3+x^2-7)$
5. $y = \dfrac{2x^2+1}{3x+1}$
6. $y = \ln \frac{x^2-5x+2}{x^2-x-8}$
7. $y = (x^2+1)^{\sin x}$
8. $y = \arctan(x^2+3x-3)$
9. $y = \sqrt{3x^2+2x-8}$

Solución:

1. $y = e^{3x^3-5x^2-2x-2} \Longrightarrow y' = (9x^2-10x-2)e^{3x^3-5x^2-2x-2}$
2. $y = \ln(5x^3+11) \Longrightarrow y' = \dfrac{15x^2}{5x^3+11}$
3. $y = (x^2-3x+1)^{17} \Longrightarrow y' = 17(x^2-3x+1)^{16}(2x-3)$
4. $y = (x^2-3x+1)(2x^3+x^2-7) \Longrightarrow y' = (2x-3)(2x^3+x^2-7)+(x^2-3x+1)(6x^2+2x)$
5. $y = \dfrac{2x^2+1}{3x+1} \Longrightarrow y' = \dfrac{(4x)(3x+1)-(2x^2+1)3}{(3x+1)^2}$
6. $y = \ln\dfrac{x^2-5x+2}{x^2-x-8} = \ln(x^2-5x+2)-\ln(x^2-x-8) \Longrightarrow y' = \dfrac{2x-5}{x^2-5x+2} - \dfrac{2x-1}{x^2-x-8}$
7. $y = (x^2+1)^{\sin x} \Longrightarrow y' = (x^2+1)^{\sin x}\left(\cos x \ln(x^2+1) + \sin x \frac{2x}{x^2+1}\right)$
8. $y = \arctan(x^2+3x-3) \Longrightarrow y' = \dfrac{2x+3}{1+(x^2+3x-3)^2}$

9. $y = \sqrt{3x^2+2x-8} \Longrightarrow y' = \dfrac{6x+2}{2\sqrt{3x^2+2x-8}}$

Problema 3 Calcular las rectas tangente y normal a la siguiente funciones en el punto $x = 1$:

1. $f(x) = \dfrac{x^2-2x+3}{x^2+2}$.

2. $f(x) = (x+1)e^{x-1}$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = 2/3$ e $y - b = m(x-a)$

$$f'(x) = -\frac{2(x^2-x-2)}{(x^2+2)^2} \Longrightarrow m = f'(1) = -\frac{4}{9}$$

Recta Tangente: $y - \frac{2}{3} = -\frac{4}{9}(x-1)$

Recta Normal: $y - \frac{2}{3} = \frac{9}{4}(x-1)$

2. $b = f(a) \Longrightarrow b = f(1) = 2$ e $y - b = m(x-a)$

$$f'(x) = (x+2)e^{x-1} \Longrightarrow m = f'(1) = 3$$

Recta Tangente: $y - 2 = 3(x-1)$

Recta Normal: $y - 2 = -\dfrac{1}{3}(x-1)$