Examen de Matemáticas 1º de Bachillerato

Enero 2017

Problema 1 Dados los números complejos $z_1 = 6 - 3i$ y $z_2 = -3 + 5i$. Se pide calcular:

a) $z_1 + z_2$ y $z_1 - z_2$

b) $z_1 \cdot z_2$

c) $\dfrac{z_1}{z_2}$

Solución:

a) $z_1 + z_2 = 3 + 2i$ y $z_1 - z_2 = 9 - 8i$

b) $z_1 \cdot z_2 = -3 + 39i$

c) $\dfrac{z_1}{z_2} = -\dfrac{33}{34} - \dfrac{21}{34}i$

Problema 2 Resolver la siguiente ecuación de segundo grado:

$$z^2 - 3z + 7 = 0$$

Solución:

$$z^2 - 3z + 7 = 0 \Longrightarrow z = \left\{ \begin{array}{l} \frac{3}{2} + \frac{\sqrt{19}}{2}i \\ \frac{3}{2} - \frac{\sqrt{19}}{2}i \end{array} \right.$$

Problema 3 Si $z = 3 - 8i$ calcular z^{10}.

Solución:

$$z = 3 - 8i = \sqrt{73}_{290^{\text{o}}33'22''} = \sqrt{73}(\cos 290^{\text{o}}33'22'' + i \sin 290^{\text{o}}33'22'')$$

$$z^{10} = (3 - 8i)^{10} = 73^5_{10\cdot 290^{\text{o}}33'22''} = 73^5_{2905^{\text{o}}33'40''} = 73^5_{25^{\text{o}}33'40''} =$$
$$73^5(\cos 25^{\text{o}}33'40'' + i \sin 25^{\text{o}}33'40'')$$

Problema 4 Resolver la ecuación $z^3 - 3i = -2$.

Solución:

$$z^3 = -2 + 3i \Longrightarrow z = \sqrt[3]{-2 + 3i}$$

$$-2 + 3i = \sqrt{13}_{123^{\text{o}}41'24''} = \sqrt{53}(\cos 123^{\text{o}}41'24'' + i \sin 123^{\text{o}}41'24'')$$

$$z = \sqrt[3]{-2 + 3i} = \left\{ \begin{array}{l} \sqrt[6]{13}_{41^{\text{o}}13'48''} = \sqrt[6]{13}(\cos 41^{\text{o}}13'48'' + i \sin 41^{\text{o}}13'48'') \\ \sqrt[6]{13}_{161^{\text{o}}13'48''} = \sqrt[6]{13}(\cos 161^{\text{o}}13'48'' + i \sin 161^{\text{o}}13'48'') \\ \sqrt[6]{13}_{281^{\text{o}}13'48''} = \sqrt[6]{13}(\cos 281^{\text{o}}13'48'' + i \sin 281^{\text{o}}13'48'') \end{array} \right.$$

Problema 5 Calcular las raices de $\sqrt[3]{7-4i}$

Solución:

$$z = 7 - 4i = \sqrt{65}_{330^{\text{o}}15'19''} = \sqrt{65}(\cos 330^{\text{o}}15'19'' + i \sin 330^{\text{o}}15'19'')$$

$$\sqrt[3]{z} = \begin{cases} \sqrt[6]{65}_{110^{\text{o}}05'06''} = \sqrt[6]{65}(\cos 110^{\text{o}}05'06'' + i \sin 110^{\text{o}}05'06'') \\ \sqrt[6]{65}_{230^{\text{o}}05'06''} = \sqrt[6]{65}(\cos 230^{\text{o}}05'06'' + i \sin 230^{\text{o}}05'06'') \\ \sqrt[6]{65}_{350^{\text{o}}05'06''} = \sqrt[6]{65}(\cos 350^{\text{o}}05'06'' + i \sin 350^{\text{o}}05'06'') \end{cases}$$