Examen de Matemáticas 1º de Bachillerato CS

Abril 2015

Problema 1 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty} \dfrac{-3x^2 - x + 7}{5x^3 + 3}$
2. $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{5x^2 - 1}{3x^2 + 8}\right)^{x^2-9}$
3. $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{x^2 + 3x + 1}{x^2 - 1}\right)^{3x}$
4. $\lim\limits_{x\longrightarrow\infty} \dfrac{\sqrt{7x^2 - 6}}{-x + 5}$
5. $\lim\limits_{x\longrightarrow 1} \dfrac{8x^4 + x^3 - 8x^2 - 3x + 2}{5x^3 + x^2 - 8x + 2}$
6. $\lim\limits_{x\longrightarrow 6} \dfrac{\sqrt{x^2 - 3} - \sqrt{5x + 3}}{x - 6}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty} \dfrac{-3x^2 - x + 7}{5x^3 + 3} = 0$
2. $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{5x^2 - 1}{3x^2 + 8}\right)^{x^2-9} = \infty$
3. $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{x^2 + 3x + 1}{x^2 - 1}\right)^{3x} = e^9$
4. $\lim\limits_{x\longrightarrow\infty} \dfrac{\sqrt{7x^2 - 6}}{-x + 5} = -\sqrt{7}$
5. $\lim\limits_{x\longrightarrow 1} \dfrac{8x^4 + x^3 - 8x^2 - 3x + 2}{5x^3 + x^2 - 8x + 2} = \dfrac{16}{9}$
6. $\lim\limits_{x\longrightarrow 6} \dfrac{\sqrt{x^2 - 3} - \sqrt{5x + 3}}{x - 6} = \dfrac{7\sqrt{33}}{66}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{6x^3+7x^2-2x+3}$

2. $y = \ln(7x^5 + 2)$

3. $y = (2x^2 - 7)^{14}$

4. $y = (3x^2 + 5x - 3)(2x^3 - 3x^2 + 1)$

5. $y = \frac{5x^2 - x + 2}{7x - 1}$

6. $y = x^9 \ln x$

Solución:

1. $y = e^{6x^3 + 7x^2 - 2x + 3} \Longrightarrow y' = (18x^2 + 14x - 2)e^{6x^3 + 7x^2 - 2x + 3}$

2. $y = \ln(7x^5 + 2) \Longrightarrow y' = \dfrac{35x^4}{7x^5 + 2}$

3. $y = (2x^2 - 7)^{14} \Longrightarrow y' = 14(2x^2 - 7)^{13}(4x)$

4. $y = (3x^2 + 5x - 3)(2x^3 - 3x^2 + 1) \Longrightarrow y' = (6x + 5)(2x^3 - 3x^2 + 1) + (3x^2 + 5x - 3)(6x^2 - 6x)$

5. $y = \dfrac{5x^2 - x + 2}{7x - 1} \Longrightarrow y' = \dfrac{(10x - 1)(7x - 1) - (5x^2 - x + 2)7}{(2x - 3)^2}$

6. $y = x^9 \ln x \Longrightarrow y' = 9x^8 \ln x + x^9 \dfrac{1}{x}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{7x^2 - 1}{x^2 - 2}$ en el punto $x = 1$.

2. $f(x) = \dfrac{3x^2 - 2}{2x + 3}$ en el punto $x = 0$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = -6$ e $y - b = m(x - a)$

$$f'(x) = -\frac{26x}{(x^2 - 2)^2} \Longrightarrow m = f'(1) = -26$$

Recta Tangente: $y + 6 = -26(x - 1)$

Recta Normal: $y + 6 = \frac{1}{26}(x - 1)$

2. $b = f(a) \Longrightarrow b = f(0) = -\dfrac{2}{3}$ e $y - b = m(x - a)$

$$f'(x) = \frac{2(3x^2 + 9x + 2)}{(2x + 3)^2} \Longrightarrow m = f'(0) = \frac{4}{9}$$

Recta Tangente: $y + \dfrac{2}{3} = \dfrac{4}{9}x$

Recta Normal: $y + \dfrac{2}{3} = -\dfrac{9}{4}x$

Problema 4 Calcular las siguientes integrales:

1. $\displaystyle\int (5x^2 - 2x + 5)\, dx$

2. $\displaystyle\int \left(\frac{5x^2 - 6x + 3}{x} - 9e^x \right) dx$

3. $\displaystyle\int \left(\frac{3x^2 - 2\sqrt[5]{x^2} + 5}{x} \right) dx$

Solución:

1. $\displaystyle\int (5x^2 - 2x + 5)\, dx = \frac{5x^3}{3} - x^2 + 5x + C$

2. $\displaystyle\int \left(\frac{5x^2 - 6x + 3}{x} - 9e^x \right) dx = \frac{5x^2}{2} - 6x + 3\ln|x| - 9e^x + C$

3. $\displaystyle\int \left(\frac{3x^2 - 2\sqrt[5]{x^2} + 5}{x} \right) dx = \frac{3x^2}{2} - 5\sqrt[5]{x^2} + 5\ln|x| + C$