Examen de Matemáticas 1º de Bachillerato CN

Mayo 2014

Problema 1 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{5x^2-3x+3}-\sqrt{5x^2+4x-2}\right)$
2. $\lim\limits_{x\longrightarrow 1}\dfrac{7x^4-5x^2+3x-5}{6x^5-x-5}$
3. $\lim\limits_{x\longrightarrow 7}\dfrac{\sqrt{x^2-8}-\sqrt{6x-1}}{x-7}$
4. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{2x^2-3x+2}{2x^2-1}\right)^{3x}$
5. $\lim\limits_{x\longrightarrow\infty}\dfrac{e^{2x^2+3}}{5x-1}$
6. $\lim\limits_{x\longrightarrow\infty}\dfrac{e^{2x-7}-15}{e^{2x+4}-3}$
7. $\lim\limits_{x\longrightarrow 0}\dfrac{\sin^2 x-2x}{5x\cos x}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{5x^2-3x+3}-\sqrt{5x^2+4x-2}\right)=-\dfrac{7\sqrt{5}}{10}$
2. $\lim\limits_{x\longrightarrow 1}\dfrac{7x^4-5x^2+3x-5}{6x^5-x-5}=\dfrac{21}{29}$
3. $\lim\limits_{x\longrightarrow 7}\dfrac{\sqrt{x^2-8}-\sqrt{6x-1}}{x-7}=\dfrac{4\sqrt{41}}{41}$
4. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{2x^2-3x+2}{2x^2-1}\right)^{3x}=e^{-9/2}$
5. $\lim\limits_{x\longrightarrow\infty}\dfrac{e^{2x^2+3}}{5x-1}=\infty$
6. $\lim\limits_{x\longrightarrow\infty}\dfrac{e^{2x-7}-15}{e^{2x+4}-3}=e^{-11}$
7. $\lim\limits_{x\longrightarrow 0}\dfrac{\sin^2 x-2x}{5x\cos x}=-\dfrac{2}{5}$

Problema 2 Calcular las siguientes derivadas:

1. $y = (5x^2 - 8)^{15}$
2. $y = \ln\left(\dfrac{11x - 7}{9x^2}\right)$
3. $y = x^5 \sec x$
4. $y = \dfrac{\cos x}{x^2 - 6}$
5. $y = \sec(4x^2 - 3x + 2)^3$
6. $y = (\sin x)^{9x+1}$

Solución:

1. $y = (5x^2 - 8)^{15} \Longrightarrow y' = 15(5x^2 - 8)^{14}(10x)$
2. $y = \ln\left(\dfrac{11x - 7}{9x^2}\right) \Longrightarrow y' = \dfrac{11}{11x - 7} - \dfrac{18x}{9x^2}$
3. $y = x^5 \sec x \Longrightarrow y' = 5x^4 \sec x + x^5 \sec x \tan x$
4. $y = \dfrac{\cos x}{x^2 - 6} \Longrightarrow y' = \dfrac{-\sin x \cdot (x^2 - 6) - (2x)\cos x}{(x^2 - 6)^2}$
5. $y = \sec(4x^2 - 3x + 2)^3 \Longrightarrow y' = 3(8x - 3)(4x^2 - 3x + 2)^2 \tan(4x^2 - 3x + 2)^3 \sec(4x^2 - 3x + 2)^3$
6. $y = (\sin x)^{9x+1} \Longrightarrow y' = (\sin x)^{9x+1}\left(9\ln(\sin x) + (9x + 1)\dfrac{\cos x}{\sin x}\right)$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{5x + 2}{x - 1}$ en el punto $x = 2$.
2. $f(x) = (x + 5)e^{x+1}$ en el punto $x = -1$.

Solución:

1. $b = f(a) \Longrightarrow b = f(2) = 12$ e $y - b = m(x - a)$

   $$f'(x) = -\frac{7}{(x - 1)^2} \Longrightarrow m = f'(2) = -7$$

   Recta Tangente: $y - 12 = -7(x - 2)$

   Recta Normal: $y - 12 = \frac{1}{7}(x - 2)$

2. $b = f(a) \Longrightarrow b = f(-1) = 4$ e $y - b = m(x - a)$

$$f'(x) = (x+6)e^{x+1} \Longrightarrow m = f'(-1) = 5$$

Recta Tangente: $y - 4 = 5(x+1)$

Recta Normal: $y - 4 = -\dfrac{1}{5}(x+1)$

Problema 4 Calcular las siguientes integrales:

1. $\displaystyle\int (2x^2 - 5x + 3)\, dx$

2. $\displaystyle\int \left(\frac{4x^2 - 3\sqrt[4]{x} - 2}{x} - 5e^x \right) dx$

3. $\displaystyle\int (x^3 - 3x + 1)\, dx$ sabiendo que la primitiva $F(1) = 1$.

4. $\displaystyle\int 7xe^{5x^2-1}\, dx$

5. $\displaystyle\int \frac{3x}{8x^2 + 2}\, dx$

Solución:

1. $\displaystyle\int (2x^2 - 5x + 3)\, dx = \frac{2x^3}{3} - \frac{5x^2}{2} + 3x + C$

2. $\displaystyle\int \left(\frac{3x^2 - 2\sqrt[4]{x} - 4}{x} - 5e^x \right) dx = 2x^2 - 12x^{1/4} - 2\ln|x| - 5e^x + C$

3. $$F(x) = \int (x^3 - 3x + 1)\, dx = \frac{x^4}{4} - \frac{3x^2}{2} + x + C$$

   como
   $$F(1) = 1 \Longrightarrow \frac{1}{4} - \frac{3}{2} + 1 + C = 1 \Longrightarrow C = \frac{5}{4}$$
   $$F(x) = \frac{x^4}{4} + \frac{3x^2}{2} + x + \frac{5}{4}$$

4. $\displaystyle\int 7xe^{5x^2-1}\, dx = \frac{7}{10}e^{5x^2-1} + C$

5. $\displaystyle\int \frac{3x}{8x^2 + 2}\, dx = \frac{3}{16}\ln|8x^2 + 2| + C$