Examen de Matemáticas 1º de Bachillerato
Abril 2011

Problema 1 Calcular los siguientes límites

1. $\lim\limits_{x \longrightarrow \infty} \left(\sqrt{x^2 - x + 1} - \sqrt{x^2 + 5x - 1} \right)$

2. $\lim\limits_{x \longrightarrow \infty} \left(\dfrac{x + 5}{x - 1} \right)^{x^2}$

3. $\lim\limits_{x \longrightarrow \infty} \left(\dfrac{3x + 1}{3x} \right)^{x+2}$

4. $\lim\limits_{x \longrightarrow 0} \dfrac{x^2 + 1 - \cos x}{x \sin x}$

5. $\lim\limits_{x \longrightarrow 1} \dfrac{3x^5 + 2x^3 - 7x^2 + 2}{x^3 + 3x^2 - 5x + 1}$

Solución:

1. $\lim\limits_{x \longrightarrow \infty} \left(\sqrt{x^2 - x + 1} - \sqrt{x^2 + 5x - 1} \right) = -3$

2. $\lim\limits_{x \longrightarrow \infty} \left(\dfrac{x + 5}{x - 1} \right)^{x^2} = \infty$

3. $\lim\limits_{x \longrightarrow \infty} \left(\dfrac{3x + 1}{3x} \right)^{x+2} = \dfrac{1}{3}$

4. $\lim\limits_{x \longrightarrow 0} \dfrac{x^2 + 1 - \cos x}{x \sin x} = \dfrac{3}{2}$

5. $\lim\limits_{x \longrightarrow 1} \dfrac{3x^5 + 2x^3 - 7x^2 + 2}{x^3 + 3x^2 - 5x + 1} = \dfrac{7}{4}$

Problema 2 Calcular la integral de las siguientes funciones

1. $\displaystyle\int \left(\dfrac{x^2 + x - 1}{x^2} \right) dx$

2. $\displaystyle\int \left(5x e^{x^2 + 3} \right) dx$

3. $\displaystyle\int x \cos x \, dx$

4. $\displaystyle\int x e^x \, dx$

5. $\displaystyle\int \frac{x^3}{x^2 + x - 2}\, dx$

Solución:

1. $\displaystyle\int \left(\frac{x^2 + x - 1}{x^2} \right) dx = x + \ln|x| + \frac{1}{x} + C$

2. $\displaystyle\int \left(5x e^{x^2+3} \right) dx = \frac{5}{2} e^{x^2+3}$

3. $\displaystyle\int x \cos x \, dx = x \sin x + \cos x + C$

4. $\displaystyle\int x e^x \, dx = e^x (x - 1) + C$

5. $\displaystyle\int \frac{x^3}{x^2 + x - 2}\, dx = \frac{x^2}{2} - x + \frac{1}{3} \ln|x - 1| + \frac{8}{3} \ln|x + 2| + C$