Examen de Matemáticas 1º de Bachillerato

Diciembre 2010

Problema 1 Calcular los siguientes límites

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+3x-2}\right)$
2. Calcular n que cumpla:
$$\lim_{x\longrightarrow\infty}\left(\frac{x^2+2x+1}{x^2+x-1}\right)^{nx}=7$$
3. $\lim\limits_{x\longrightarrow 2}\dfrac{\sqrt{x^2+5}-\sqrt{4x+1}}{x-2}$
4. $\lim\limits_{x\longrightarrow 3}\dfrac{x^3-2x^2+x-12}{x^3-5x-12}$
5. $\lim\limits_{x\longrightarrow\infty}\dfrac{xe^x-1}{e^x+2}$
6. $\lim\limits_{x\longrightarrow 0}\dfrac{x\sin x}{1-\cos x}$
7. $\lim\limits_{x\longrightarrow 0}\dfrac{\sin^2 x}{2x^2+\sin x}$
8. $\lim\limits_{x\longrightarrow \pi}\dfrac{1+\cos x}{\sin^2 x}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+3x-2}\right)=-1$
2. $$\lim_{x\longrightarrow\infty}\left(\frac{x^2+2x+1}{x^2+x-1}\right)^{nx}=7\Longrightarrow n=\ln 7$$
3. $\lim\limits_{x\longrightarrow 2}\dfrac{\sqrt{x^2+5}-\sqrt{4x+1}}{x-2}=0$
4. $\lim\limits_{x\longrightarrow 3}\dfrac{x^3-2x^2+x-12}{x^3-5x-12}=\dfrac{8}{11}$
5. $\lim\limits_{x\longrightarrow\infty}\dfrac{xe^x-1}{e^x+2}=\infty$
6. $\lim\limits_{x\longrightarrow 0}\dfrac{x\sin x}{1-\cos x}=2$

7. $\lim\limits_{x \longrightarrow 0} \dfrac{\sin^2 x}{2x^2 + \sin x} = 0$

8. $\lim\limits_{x \longrightarrow \pi} \dfrac{1 + \cos x}{\sin^2 x} = \dfrac{1}{2}$

Problema 2 Calcular la derivada de las siguientes funciones

1. $y = (x^2 + x - 1)^{12}$
2. $y = (\sin x) \ln x$
3. $y = 2x \tan x$
4. $y = \ln\left(\dfrac{x^2+5}{x^2-2}\right)$
5. $y = \arctan(x^2 + 2)$
6. $y = 7^{x^2+5}$
7. $y = e^x \cos 2x$
8. $y = \dfrac{\sin x}{x^2+1}$
9. $y = (x^2 - 1)^{\sin x}$

Solución:

1. $y = (x^2 + x - 1)^{12} \Longrightarrow y' = 12(x^2 + x - 1)^{11}(2x + 1)$
2. $y = (\sin x) \ln x \Longrightarrow y' = \cos x \ln x + \dfrac{\sin x}{x}$
3. $y = 2x \tan x \Longrightarrow y' = 2 \tan x + \dfrac{2x}{\cos^2 x}$
4. $y = \ln\left(\dfrac{x^2+5}{x^2-2}\right) \Longrightarrow y' = \dfrac{2x}{x^2+5} - \dfrac{2x}{x^2-2}$
5. $y = \arctan(x^2 + 2) \Longrightarrow y' = \dfrac{2x}{1 + (x^2+2)^2}$
6. $y = 7^{x^2+5} \Longrightarrow y' = 2x7^{x^2+5} \ln 7$
7. $y = e^x \cos 2x \Longrightarrow y' = e^x \cos x - 2e^x \sin x$
8. $y = \dfrac{\sin x}{x^2+1} \Longrightarrow y' = \dfrac{\cos x(x^2+1) - 2x \sin x}{(x^2+1)^2}$
9. $y = (x^2 - 1)^{\sin x} \Longrightarrow y' = (x^2 - 1)^{\sin x} \left(\cos x \ln(x^2 - 1) + \dfrac{2x \sin x}{x^2 - 1}\right)$

Problema 3 Calcular las rectas tangente y normal de la siguiente función en el punto de abcisa $x = 1$

$$f(x) = \frac{2x^2}{x^2+1}$$

Solución:

$f(1) = 1,\ f'(x) = \dfrac{4x}{(x^2+2)^2} \Longrightarrow m = f'(1) = 1$

Recta tangente: $y - 1 = x - 1 \Longrightarrow y = x$

Recta normal: $y - 1 = -(x - 1) \Longrightarrow y = -x + 2$