Examen de Matemáticas 1º de Bachillerato
Junio 2011

Problema 1 Calcular los siguientes límites

1. $\lim\limits_{x \longrightarrow \infty} \left(\sqrt{4x^2 - x + 1} - \sqrt{4x^2 + 2} \right)$

2. Calcular n sabiendo que $\lim\limits_{x \longrightarrow \infty} \left(\dfrac{3x+5}{3x-1} \right)^{2nx} = 3$

3. $\lim\limits_{x \longrightarrow \infty} \dfrac{2e^x - x^2}{3e^x}$

4. $\lim\limits_{x \longrightarrow 0} \dfrac{\ln(\cos x)}{\ln(\sin x + 1)}$

5. $\lim\limits_{x \longrightarrow \infty} \dfrac{\sqrt{5x^4 - 2x + 1}}{2x^2 - 2}$

Solución:

1. $\lim\limits_{x \longrightarrow \infty} \left(\sqrt{4x^2 - x + 1} - \sqrt{4x^2 + 2} \right) = -\dfrac{1}{4}$

2. Calcular n sabiendo que $\lim\limits_{x \longrightarrow \infty} \left(\dfrac{3x+5}{3x-1} \right)^{2nx} = 3 \Longrightarrow n = \dfrac{\ln 3}{4}$

3. $\lim\limits_{x \longrightarrow \infty} \dfrac{2e^x - x^2}{3e^x} = \dfrac{2}{3}$

4. $\lim\limits_{x \longrightarrow 0} \dfrac{\ln(\cos x)}{\ln(\sin x + 1)} = 0$

5. $\lim\limits_{x \longrightarrow \infty} \dfrac{\sqrt{5x^4 - 2x + 1}}{2x^2 - 2} = \dfrac{\sqrt{5}}{2}$

Problema 2 Calcular la integral de las siguientes funciones

1. $\displaystyle\int \left(\dfrac{3x - 2\sqrt{x} + 1}{x^2} \right) dx$

2. $\displaystyle\int \dfrac{5x}{\cos^2(x^2 - 1)} \, dx$

3. $\displaystyle\int x^2 \ln x \, dx$

4. $\displaystyle\int x \sin 2x \, dx$

5. $\displaystyle\int \frac{x^2+1}{x^2+x-2}\,dx$

Solución:

1. $\displaystyle\int \left(\frac{3x-2\sqrt{x}+1}{x^2}\right)dx = 3\ln|x| + \frac{4}{\sqrt{x}} - \frac{1}{x} + C$

2. $\displaystyle\int \frac{5x}{\cos^2(x^2-1)}\,dx = \frac{5}{2}\tan(x^2-1) + C$

3. $\displaystyle\int x^2\ln x\,dx = \frac{x^3\ln x}{3} - \frac{x^3}{9} + C$

4. $\displaystyle\int x\sin 2x\,dx = -\frac{x\cos 2x}{2} + \frac{\sin 2x}{4} + C$

5. $\displaystyle\int \frac{x^2+1}{x^2+x-2}\,dx = x + \frac{2\ln|x-1|}{3} - \frac{5\ln|x+2|}{3} + C$