

Examen de Matemáticas 1º de Bachillerato
Octubre 2009

Problema 1 Discutir y resolver por el método de Gauss los siguientes sistemas:

$$\left\{ \begin{array}{l} x - y = 2 \\ -x + 2y + 3z = -1 \\ x + y + 6z = 4 \end{array} \right. ; \left\{ \begin{array}{l} x - 2y + z = 1 \\ y - z = 1 \\ 3x + 2y + z = -1 \end{array} \right.$$

Solución:

$$\left\{ \begin{array}{l} x - y = 2 \\ -x + 2y + 3z = -1 \\ x + y + 6z = 4 \end{array} \right. \text{ Sistema Compatible Indeterminado} \implies \left\{ \begin{array}{l} x = 3 - 3\lambda \\ y = 1 - 3\lambda \\ z = \lambda \end{array} \right.$$

$$\left\{ \begin{array}{l} x - 2y + z = 1 \\ y - z = 1 \\ 3x + 2y + z = -1 \end{array} \right. \text{ Sistema Compatible Determinado} \implies \left\{ \begin{array}{l} x = 1 \\ y = -1 \\ z = -2 \end{array} \right.$$

Problema 2 Resolver las ecuaciones:

- $\log(2 - x) - \log(x - 1) = 2$
- $\log(3 - x^2) - \log x = 1 + \log(x + 2)$
- $4^{x^2-1} \cdot 2^{x+5} = 32^{x+1}$
- $4^{x-2} - 2^{x+1} - 3 = 0$

Solución:

$$1. \log(2 - x) - \log(x - 1) = 2 \implies \ln \frac{2 - x}{x - 1} = \log 100 \implies$$

$$101x = 102 \implies x = \frac{102}{101}.$$

$$2. \log(3 - x^2) - \log x = 1 + \log(x + 2) \implies \log \frac{3 - x^2}{x} = \log 10(x + 2) \implies$$
$$11x^2 + 20x - 3 = 0 \implies x = 0,139, \quad x = -1,958(\text{no vale}).$$

3.

$$4^{x^2-1} \cdot 2^{x+5} = 32^{x+1} \implies x^2 - 4x - 2 = 0 \implies \left\{ \begin{array}{l} x = 2,4142 \\ x = -0,414 \end{array} \right.$$

4.

$$4^{x-2} - 2^{x+1} - 3 = 0 \implies t^2 - 32t - 48 = 0 \implies \left\{ \begin{array}{l} t = 33,435 \implies x = 5,063 \\ t = -1,435 \text{ no vale} \end{array} \right.$$

Problema 3 Resolver los siguientes sistemas:

$$\begin{cases} x \cdot y = 2 \\ 3x + 2y = 7 \end{cases} ; \begin{cases} x^2 + 2y^2 = 6 \\ x - 2y = 0 \end{cases}$$

Solución:

$$\begin{cases} x \cdot y = 2 \\ 3x + 2y = 7 \end{cases} \implies \begin{cases} x = 1, y = 2 \\ x = 4/3, y = 3/2 \end{cases}$$

$$\begin{cases} x^2 + 2y^2 = 6 \\ x - 2y = 0 \end{cases} \implies \begin{cases} x = 2, y = 1 \\ x = 2, y = -1 \end{cases}$$

Problema 4 Resolver las inecuaciones siguientes:

1. $\frac{2x+1}{3} - \frac{x-1}{6} \leq 1 + \frac{2x+3}{2}$

2. $\frac{x^2+2x-15}{x^2+x-2} \geq 0$

3. $\frac{x^2-x-6}{x^2+x-2} \leq 0$

Solución:

1. $\frac{2x+1}{3} - \frac{x-1}{6} \leq 1 + \frac{2x+3}{2} \implies [-4, \infty)$

2. $\frac{x^2+2x-15}{x^2+x-2} \geq 0 \implies (-\infty, -5] \cup (-2, 1) \cup [4, \infty)$

3. $\frac{x^2-x-6}{x^2+x-2} \leq 0 \implies (-\infty, -2) \cup (-2, 1) \cup [3, \infty)$