Examen de Matemáticas 1º de Bachillerato

Marzo 2009

Problema 1 Calcular los siguientes límites

1. $\lim_{x\longrightarrow\infty}\left(\frac{2x-1}{2x}\right)^{3x}$
2. $\lim_{x\longrightarrow\infty}\frac{3x^2-5x-2}{x^3-3x^2+x+2}$
3. $\lim_{x\longrightarrow 1}\frac{\sqrt{3x-1}-\sqrt{2x^2}}{x-1}$
4. $\lim_{x\longrightarrow 0}\frac{x\cos x}{x+\sin x}$
5. $\lim_{x\longrightarrow\infty}(\sqrt{x^2+1}-\sqrt{x^2+x-1})$
6. $\lim_{x\longrightarrow\infty}\frac{\sqrt{3x^4+x-1}}{-x^2+2}$

Solución:

1. $\lim_{x\longrightarrow\infty}\left(\frac{2x-1}{2x}\right)^{3x}=e^{-3/2}$
2. $\lim_{x\longrightarrow\infty}\frac{3x^2-5x-2}{x^3-3x^2+x+2}=0$
3. $\lim_{x\longrightarrow 1}\frac{\sqrt{3x-1}-\sqrt{2x^2}}{x-1}=-\frac{\sqrt{2}}{4}$
4. $\lim_{x\longrightarrow 0}\frac{x\cos x}{x+\sin x}=\frac{1}{2}$
5. $\lim_{x\longrightarrow\infty}(\sqrt{x^2+1}-\sqrt{x^2+x-1})=-\frac{1}{2}$
6. $\lim_{x\longrightarrow\infty}\frac{\sqrt{3x^4+x-1}}{-x^2+2}=-\sqrt{3}$

Problema 2 Calcular la derivada de las siguientes funciones

1. $y=e^x\csc(x^2+1)$
2. $y=(x^2+1)^{\sin x}$
3. $y=\ln\frac{\sin x}{x+1}$

4. $y = e^{x+1} \cos x$
5. $y = \sin^{10}(x^2 + 1)$
6. $y = \dfrac{x^2}{\arctan x}$

Solución:

1. $y' = e^x \csc(x^2 + 1) - e^x \cot x \csc x$
2. $y' = (x^2 + 1)^{\sin x}(\ln(x^2 + 1) \cos x + \frac{2x}{x^2+1} \sin x)$
3. $y' = \dfrac{\cos x}{\sin x} - \dfrac{1}{x + 1}$
4. $y' = e^{x+1} \cos x - e^{x+1} \sin x$
5. $y' = 20x \sin^9(x^2 + 1) \cos(x^2 + 1)$
6. $y' = \dfrac{2x \arctan x - \frac{x^2}{1+x^2}}{(\arctan x)^2}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones

1. $f(x) = \dfrac{e^x + 1}{x}$ en $x = 1$
2. $f(x) = x^2 \sin x$ en $x = \dfrac{\pi}{2}$

Solución:

1. $f(1) = e + 1,\ f'(x) = \dfrac{xe^x - e^x - 1}{x^2} \Longrightarrow m = f'(1) = -1$

   Recta tangente: $y - e - 1 = -(x - 1)$

   Recta normal: $y - e - 1 = (x - 1)$

2. $f\left(\dfrac{\pi}{2}\right) = \left(\dfrac{\pi}{2}\right)^2,\ f'(x) = 2x \sin x + x^2 \cos x \Longrightarrow f'\left(\dfrac{\pi}{2}\right) = 2\left(\dfrac{\pi}{2}\right) = \pi$

   Recta tangente: $y - \left(\dfrac{\pi}{2}\right)^2 = \pi\left(x - \dfrac{\pi}{2}\right)$

   Recta normal: $y - \left(\dfrac{\pi}{2}\right)^2 = -\dfrac{1}{\pi}\left(x - \dfrac{\pi}{2}\right)$

Problema 4 Calcular las inmtegrales siguientes:

1. $\displaystyle\int xe^{7x^2-1}\,dx$

2. $\displaystyle\int \frac{2x}{1+x^4}\,dx$

3. $\displaystyle\int x^4 \ln x\,dx$

4. $\displaystyle\int \frac{x^3-\sqrt{x}+5x-1}{x^2}\,dx$

Solución:

1. $\displaystyle\int xe^{7x^2-1}\,dx = \frac{e^{7x^2-1}}{14} + C$

2. $\displaystyle\int \frac{2x}{1+x^4}\,dx = \arctan(x^2) + C$

3. $\displaystyle\int x^4 \ln x\,dx = \frac{x^5(5\ln x - 1)}{25} + C$

4. $\displaystyle\int \frac{x^3-\sqrt{x}+5x-1}{x^2}\,dx = \frac{x^2}{2} + \frac{2}{\sqrt{x}} + \frac{1}{x} + 5\ln x + C$