Examen de Matemáticas 1º de Bachillerato
Junio 2006

Problema 1 Calcular:

1. $(2 - 4\,i)(1 + i)$

2. $\dfrac{3 - i}{5 + 2\,i}$

3. $(4 + i)^2$

Solución:

1. $(2 - 4\,i)(1 + i) = 6 - 2i$

2. $\dfrac{3 - i}{5 + 2\,i} = \dfrac{13}{29} - \dfrac{11}{29}\,i$

3. $(4 + i)^2 = 15 + 8\,i$

Problema 2 calcular y pasar a forma polar y paramétrica

1. $\dfrac{2_{60º}}{3_{15º}}$

2. $4_{40º} \cdot 3_{80º}$

3. $(5_{40º})^5$

Solución:

1. $\dfrac{2_{60º}}{3_{15º}} = \left(\dfrac{2}{3}\right)_{45º} = \dfrac{2}{3}(\cos 45º + i\,\sin 45º) = \dfrac{\sqrt{2}}{3} + \dfrac{\sqrt{2}}{3}\,i$

2. $4_{40º} \cdot 3_{80º} = 12_{120º} = 12(cos 120º + i\,\sin 120º) = -6 + 6\sqrt{3}\,i$

3. $(5_{40º})^5 = 5^5_{200º} = 5^5(\cos 200º + i\,\sin 200º) = -2936,5 - 1068,8\,i$

Problema 3 Calcular $\sqrt[4]{z}$ donde $z = 1 - 3\,i$

Solución:

$$z = 1 - 3\,i = \sqrt{10}_{288,43}$$

$$\sqrt[4]{1 - 3\,i} = (\sqrt{10})^{1/4}_{\frac{288,43 + k\cdot 360º}{4}}$$

$$\begin{cases} k = 0 \Longrightarrow 10^{1/8}_{72,11º} = 10^{1/8}(\cos 72,11º + i\,\sin 72,11º) = 0,409 + 1,27\,i \\ k = 1 \Longrightarrow 10^{1/8}_{162,11º} = 10^{1/8}(\cos 162,11º + i\,\sin 162,11º) = -1,27 + 0,41\,i \\ k = 2 \Longrightarrow 10^{1/8}_{252,11º} = 10^{1/8}(\cos 252,11º + i\,\sin 252,11º) = -0,41 - 1,27\,i \\ k = 3 \Longrightarrow 10^{1/8}_{342,11º} = 10^{1/8}(\cos 342,11º + i\,\sin 342,11º) = 1,27 - 0,41\,i \end{cases}$$

Problema 4 Resolver la ecuación: $z^2 - z + 2 = 0$

Solución:

$$z^2 - z + 2 = 0 \implies z = \frac{1 \pm \sqrt{-7}}{2} = \frac{1}{2} \pm \frac{\sqrt{7}}{2}\, i$$

Problema 5 Resolver la ecuación: $z^3 + 1 = 0$.

Solución:

$$z^3 = -1 \implies z = \sqrt[3]{-1} = \sqrt[3]{1_{180^\mathrm{o}}} = 1_{\frac{180^\mathrm{o} + k \cdot 360^\mathrm{o}}{3}}$$

$$\begin{cases} k = 0 \implies 1_{60^\mathrm{o}} = \cos 60^\mathrm{o} + i \, \sin 60^\mathrm{o} = \frac{1}{2} + \frac{\sqrt{3}}{2}\, i \\ k = 1 \implies 1_{180^\mathrm{o}} = \cos 180^\mathrm{o} + i \, \sin 180^\mathrm{o}) = -1 \\ k = 2 \implies 1_{300^\mathrm{o}} = \cos 300^\mathrm{o} + i \, \sin 300^\mathrm{o}) = \frac{1}{2} - \frac{\sqrt{3}}{2}\, i \end{cases}$$