

Examen de Matemáticas 1º de Bachillerato
Octubre 2005

Problema 1 Discutir y resolver por el método de Gauss los siguientes sistemas:

$$\begin{cases} x+ & 2y- & z = 1 \\ 2x- & y- & z = 0 \\ 3x+ & y+ & 2z = 2 \end{cases} ; \begin{cases} x+ & y & + & z = 1 \\ x+ & 2y & - & z = 2 \\ 2x+ & 3y & & = 4 \end{cases}$$

Solución:

$$\begin{cases} x+ & 2y- & z = 1 \\ 2x- & y- & z = 0 \\ 3x+ & y+ & 2z = 2 \end{cases} \text{ Sistema Compatible Determinado} \implies \begin{cases} x = 7/20 \\ y = 9/20 \\ z = 1/4 \end{cases}$$

$$\begin{cases} x+ & y & + & z = 1 \\ x+ & 2y & - & z = 2 \\ 2x+ & 3y & & = 4 \end{cases} \text{ Sistema Incompatible}$$

Problema 2 Resolver las ecuaciones:

- $\ln x - \ln(1-x) = 1$
- $\log(7-x^2) - \log x = 1$
- $\log(2x+3) + \log x = -1$

Solución:

1. $\ln x - \ln(1-x) = 1 \implies \ln \frac{x}{1-x} = \ln e \implies$

$$x = e - ex \implies x = \frac{e}{e+1} = 0,7310585786.$$

2. $\log(7-x^2) - \log x = 1 \implies \log \frac{7-x^2}{x} = \log 10 \implies x^2 + 10x - 7 = 0 \implies x = 0,6568542494, x = -10,65685424$ (no vale).

3. $\log(2x+3) + \log x = -1 \implies \log(2x^2+3x) = \log 10^{-1} \implies 20x^2 + 30x - 1 = 0 \implies x = 0,03262379212; x = -1,532623792$ (no vale).

Problema 3 Resolver el siguiente sistema

$$\begin{cases} x^2 + y^2 = 1 \\ x + y = 0 \end{cases}$$

Solución:

$$\begin{cases} x^2 + y^2 = 1 \\ x + y = 0 \end{cases} \implies \begin{cases} x = \frac{\sqrt{2}}{2}, y = -\frac{\sqrt{2}}{2} \\ x = -\frac{\sqrt{2}}{2}, y = \frac{\sqrt{2}}{2} \end{cases}$$

Problema 4 Resolver las inecuaciones siguientes:

1. $\frac{2x-1}{3} + \frac{x+1}{2} \geq 1 - \frac{x}{2}$

2. $\frac{x^2+x-2}{x^2+2x-15} \leq 0$

Solución:

1. $\frac{2x-1}{3} + \frac{x+1}{2} \geq 1 - \frac{x}{2} \implies \left[\frac{1}{2}, +\infty\right)$

2. $\frac{x^2+x-2}{x^2+2x-15} \leq 0 \implies (-5, -2] \cup [1, 3)$

Problema 5 Calcular los siguientes límites:

1. $\lim_{x \rightarrow \infty} \frac{2x^3+x-1}{3x^3+2}$

2. $\lim_{x \rightarrow \infty} \frac{3x^2+2x+1}{x^5+2}$

3. $\lim_{x \rightarrow \infty} \frac{x^6+x-1}{x^4+2}$

4. $\lim_{x \rightarrow \infty} \left(\frac{2x^2+x}{x^2-1}\right)^{3x^2-1}$

5. $\lim_{x \rightarrow \infty} \left(\frac{x^2-x-1}{2x^2}\right)^{2x-1}$

6. $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x}\right)^{2x}$

Solución:

1. $\lim_{x \rightarrow \infty} \frac{2x^3+x-1}{3x^3+2} = \frac{2}{3}$

2. $\lim_{x \rightarrow \infty} \frac{3x^2+2x+1}{x^5+2} = 0$

3. $\lim_{x \rightarrow \infty} \frac{x^6+x-1}{x^4+2} = \infty$

4. $\lim_{x \rightarrow \infty} \left(\frac{2x^2+x}{x^2-1}\right)^{3x^2-1} = +\infty$

$$5. \lim_{x \rightarrow \infty} \left(\frac{x^2 - x - 1}{2x^2} \right)^{2x-1} = 0$$

$$6. \lim_{x \rightarrow \infty} \left(\frac{x+1}{x} \right)^{2x} = e^2$$

Problema 6 Calcular los siguientes límites:

$$1. \lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + 2x - 1}{x^4 - x^3 + 2x - 2}$$

$$2. \lim_{x \rightarrow \infty} (\sqrt{x-1} - \sqrt{x+1})$$

$$3. \lim_{x \rightarrow 2} \frac{\sqrt{2x^2+1} - 3}{x-2}$$

$$4. \lim_{x \rightarrow -2} \frac{x^4 + 2x^3 - x^2 - x + 2}{x^3 + 2x^2 - x - 2}$$

$$5. \lim_{x \rightarrow \infty} \frac{\sqrt{2x^2+x+2} - x}{x-1}$$

$$6. \lim_{x \rightarrow 5} \frac{2\sqrt{x-1} - 2}{x-2}$$

Solución:

$$1. \lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + 2x - 1}{x^4 - x^3 + 2x - 2} = \frac{1}{3}$$

$$2. \lim_{x \rightarrow \infty} (\sqrt{x-1} - \sqrt{x+1}) = 0$$

$$3. \lim_{x \rightarrow 2} \frac{\sqrt{2x^2+1} - 3}{x-2} = \frac{4}{3}$$

$$4. \lim_{x \rightarrow -2} \frac{x^4 + 2x^3 - x^2 - x + 2}{x^3 + 2x^2 - x - 2} = -\frac{5}{3}$$

$$5. \lim_{x \rightarrow \infty} \frac{\sqrt{2x^2+x+2} - x}{x-1} = \sqrt{2} - 1$$

$$6. \lim_{x \rightarrow 5} \frac{2\sqrt{x-1} - 2}{x-2} = 0$$