Examen de Matemáticas 1º de Bachillerato

Noviembre 2005

Problema 1 Discutir y resolver por el método de Gauss los siguientes sistemas:

$$\left\{\begin{array}{rrr} x- & y+ & z=1 \\ 2x+ & y- & 2z=2 \\ x+ & 2y- & 3z=1 \end{array}\right. \; ; \quad \left\{\begin{array}{rrr} x+ & y+ & z=1 \\ 2x+ & y- & z=2 \\ 2x+ & & z=3 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrr} x- & y+ & z=1 \\ 2x+ & y- & 2z=2 \\ x+ & 2y- & 3z=1 \end{array}\right. \text{ Sistema Compatible Indeterminado} \Longrightarrow \left\{\begin{array}{l} x=1+\frac{1}{3}z \\ y=\frac{4}{3}z \\ z=z \end{array}\right.$$

$$\left\{\begin{array}{rrr} x+ & y+ & z=1 \\ 2x+ & y- & z=2 \\ x+ & & z=3 \end{array}\right. \text{ Sistema Compatible Determinado} \Longrightarrow \left\{\begin{array}{l} x=7/5 \\ y=-3/5 \\ z=1/5 \end{array}\right.$$

Problema 2 Resolver las siguientes ecuaciones e inecuaciones:

1. $\ln(2-x)-\ln(x+2)=1$
2. $2^{2x-1}-2^x-1=0$
3. $\dfrac{2x-5}{x^2-4x-21}-1=\dfrac{x}{x+3}-\dfrac{2}{7-x}$
4. $\dfrac{x^2-4x-21}{x+1}\le 0$
5. $\sqrt{x+3}+\sqrt{x}=2$

Solución:

1. $\ln(2-x)-\ln(x+2)=1 \Longrightarrow x=\dfrac{2(1-e)}{1+e}$
2. $2^{2x-1}-2^x-1=0 \Longrightarrow x=0,4499$
3. $\dfrac{2x-5}{x^2-4x-21}-1=\dfrac{x}{x+3}-\dfrac{2}{7-x} \Longrightarrow x=6,2943, \quad x=-0,79436$
4. $\dfrac{x^2-4x-21}{x+1}\le 0 \Longrightarrow (-\infty,-3]\cup(-1,7]$
5. $\sqrt{x+3}+\sqrt{x}=2 \Longrightarrow x=\dfrac{1}{16}$

Problema 3 Calcular los siguientes límites:

1. $\lim_{x \longrightarrow 1} \dfrac{x^5 + x - 2}{2x^3 - x - 1}$

2. $\lim_{x \longrightarrow -\infty} \dfrac{x^3 + 3x - 1}{-x^2 + 2}$

3. $\lim_{x \longrightarrow \infty} \left(\dfrac{2x + 1}{2x}\right)^{x-3}$

4. $\lim_{x \longrightarrow 1} \dfrac{\sqrt{2x^2 - 1} - \sqrt{2x - 1}}{x - 1}$

Solución:

1. $\lim_{x \longrightarrow 1} \dfrac{x^5 + x - 2}{2x^3 - x - 1} = \dfrac{6}{5}$

2. $\lim_{x \longrightarrow -\infty} \dfrac{x^3 + 3x - 1}{-x^2 + 2} = +\infty$

3. $\lim_{x \longrightarrow \infty} \left(\dfrac{2x + 1}{2x}\right)^{x-3} = e^{1/2}$

4. $\lim_{x \longrightarrow 1} \dfrac{\sqrt{2x^2 - 1} - \sqrt{2x - 1}}{x - 1} = 1$

Problema 4 Calcular las derivadas de las siguientes funciones:

1. $y = (x^2 - x)^{10}(x + 1)^5$

2. $y = \dfrac{3x^2 - x - 1}{x + 2}$

3. $y = (3x - 1)^{2x+1}$

4. $y = \ln \sqrt{\dfrac{2x + 1}{x + 3}}$

5. $y = e^{\sqrt{x-2}}$

6. $y = x^{\sqrt{x}}$

Solución:

1. $y = (x^2 - x)^{10}(x + 1)^5 \Longrightarrow$

$$y' = 10(x^2 - x)^9(2x - 1)(x + 1)^5 + 5(x^2 + x)^{10}(x + 1)^4$$

2. $y = \dfrac{3x^2 - x - 1}{x + 2} \Longrightarrow y' = \dfrac{3x^2 + 12x - 1}{(x + 2)^2}$

3. $y = (3x - 1)^{2x+1} \Longrightarrow y' = (3x - 1)^{2x+1} \left[2\ln(3x - 1) + \frac{3(2x+1)}{3x+1}\right]$

4. $y = \ln\sqrt{\dfrac{2x+1}{x+3}} \Longrightarrow y' = \dfrac{1}{2}\ln\sqrt{\dfrac{2x+1}{x+3}}\left[\dfrac{2}{2x+1} - \dfrac{1}{x+3}\right]$

5. $y = e^{\sqrt{x-2}} \Longrightarrow y' = \dfrac{1}{2\sqrt{x-2}}\, e^{\sqrt{x-2}}$

6. $y = x^{\sqrt{x}} \Longrightarrow y' = x^{\sqrt{x}}\left[\dfrac{\ln x}{2\sqrt{x}} + \dfrac{\sqrt{x}}{x}\right]$

Problema 5 Calcular las rectas tangente y normal a la función $f(x) = \dfrac{3x-1}{x+2}$ en el punto de abcisa $x = 1$.

Solución:

$$a = 1,\ \ f(a) = f(1) = \frac{2}{3}$$

$$f'(x) = \frac{7}{(x+2)^2} \Longrightarrow m = f'(1) = \frac{7}{9}$$

Recta Tangente: $y - \frac{2}{3} = \frac{7}{9}(x-1)$

Recta Normal: $y - \frac{2}{3} = -\frac{9}{7}(x-1)$